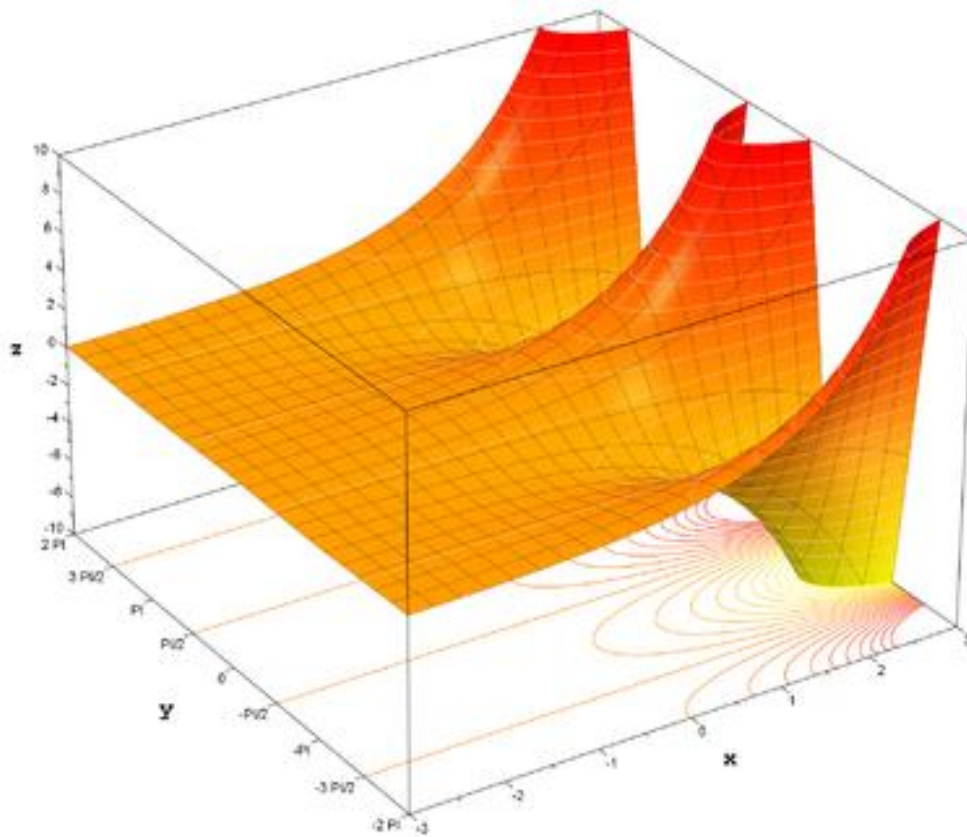


MAT12X – Intermediate Algebra

Workshop I - Exponential Functions



CHANDLER-GILBERT
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Overview

Workshop I

- Exponential Functions of the form $y = ab^x$
- Properties of the increasing and decreasing exponential functions
- Exponential Function of the Form $A = P \left(1 + \frac{r}{n} \right)^{nt}$
- Exponential Function of the Form $A = ae^{rt}$

Exponential Functions of the form $y = ab^x$

As the name of an exponential function is described, it involves an exponent. This exponent is represented with a variable rather than a constant, and its base is represented with constant value rather than a variable. Let $f(x) = ab^x$ be an exponential function where “b” is its change factor (or a constant), the exponent “x” is the independent variable (or input of the function), the coefficient “a” is called the initial value of the function (or the y-intercept), and “f(x)” represent the dependent variable (or output of the function).

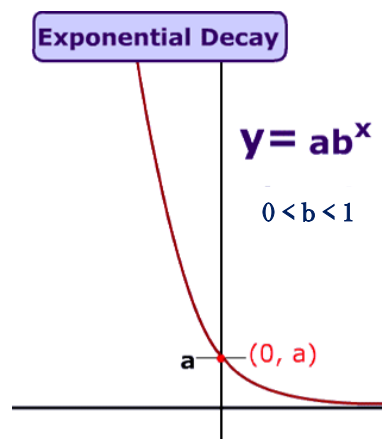
Example:

A population growth of a town is a function of time represented by $p(t) = 3^t$; identify the following

Initial value ___ Independent variable ___ Dependent variable ___ Fixed base ___

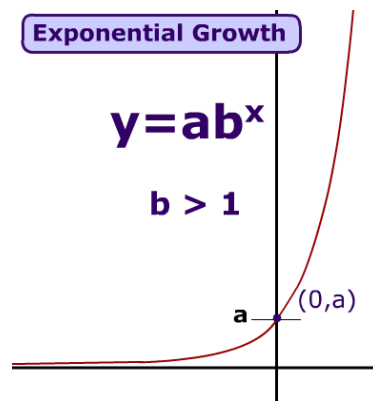
Exponential functions defined by an equation of the form $y = ab^x$ are called exponential decay functions if the change factor “b” (fixed base value) is $0 < b < 1$, or it is also called exponential growth functions if the change factor is $b > 1$.

We also can state that an exponential function is decreasing if it’s change factor “b” is $0 < b < 1$ as it is shown on its general shape in the following graph.



As we can notice on this graph, the y-intercept is (0,a) and it is located at the initial value “a”. There is no x-intercept. The domain for an exponential decay function of this form is all real numbers and the range is $y > 0$.

In addition, we can say that an exponential function is increasing if it has a change factor greater than 1 as it is shown on its general shape in the next graph



Notice that on this graph, the y-intercept is (0,a) and it is located at the initial value “a”. There is no x-intercept. The domain for an exponential growth function of this form is all real numbers and the range is $y > 0$.

Example:

Graph the function $f(x) = 2(1.5)^x$ in the empty space below, and answer the following questions by filling in the black

y-intercept _____ x-intercept _____

domain _____ range _____

Identify if the function has a decay or growth factor and its value _____

Is the graph increasing or decreasing? _____

Exponential Function of the Form $A = P \left(1 + \frac{r}{n}\right)^{nt}$

(Compound Interest Formula)

There is another form of an exponential function that is called the compound interest formula. This formula is usually used to find a current balance or compound amount “A” in an account when someone deposit certain amount in an account; this deposit amount is called the principal “P”. This formula also includes an annual interest “r” (in decimal form), and a constant “n” that represent the number of times per year that the interest is compounded, and it also includes the input or independent variable “t” that represent the time in years the money has been invested.

Example:

Suppose John has \$2000 that he invests in an account that pays 2.5% interest compounded quarterly. How much money does John have at the end of 10 years?

Solution:

First you need to identify the following data to substitute it in the compound interest formula and to be able to find the amount of money John has at the end of the 10 years.

P =

r =

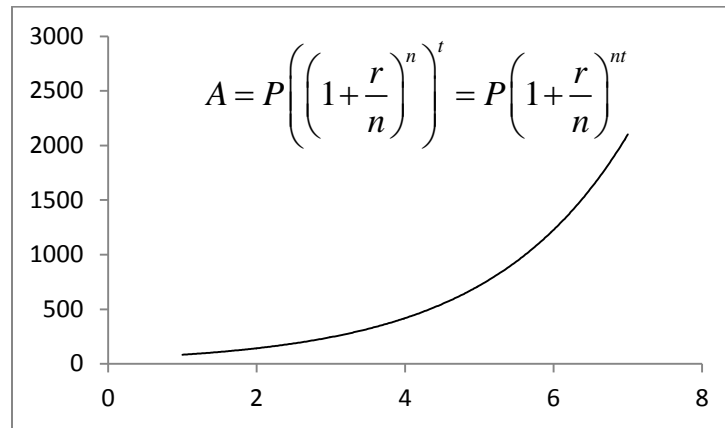
n =

t =

A =

If we compare the compound interest formula $A = P \left(1 + \frac{r}{n}\right)^{nt}$ with the general form $y = ab^x$ of the exponential function, we can see that “P” is like our initial value “a”; also notice that $\left(1 + \frac{r}{n}\right)^n$ represents the fixed base (change factor) “b”, where “r” is the growth rate; we can also identify the exponent “t” as our independent variable (input) just like “x” is in the general form, and finally “A” is like the “y” that represents the dependent variable (output).

The graph for the compound interest formula has the same shape as the general form of the exponential equation with a growth factor $b = \left(1 + \frac{r}{n}\right)^n$ as it is shown below.



Notice that the domain for both of these graphs is all real numbers and the range is all real numbers greater than zero. The y-intercept (0,1) is given by the initial value “P” when $t=0$, as it is shown on the graph, and there is no x-intercept. The graph with a decay factor is decreasing, and the graph with a growth factor is increasing. Therefore, we can conclude that the compound interest formula also has similar properties as the general form of the exponential equation due to the same shape of their graphs.

Exponential Function of the Form $A = ae^{rt}$

(Continuous Compounding Formula)

Notice that the formula $A = a(e^r)^t$, which is used when a quantity decreases or increases at a constant rate just, is just another form of the general equation of the exponential function $y = ab^x$, where $a = a$ represent the initial amount, the “ r ” represents the constant rate, the independent variable $x = t$ represents the time in years, $b = (e^r)$ represents the fixed base, “ e ” is the constant irrational number approximately equal to 2.718, and the dependent variable $y = A$ is the output amount of the investment.

Example:

If you invest \$1,000 at an annual interest rate of 5% compounded continuously, calculate the final amount you will have in the account after five years.

Solution:

We need to find the following data to be able to find the final amount in the account.

$a =$

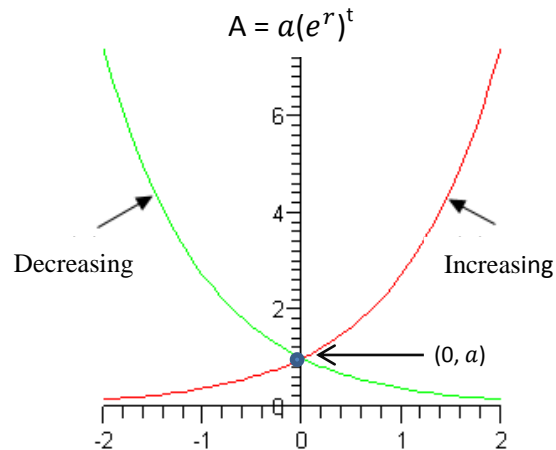
$r =$

$t =$

$A =$

Since the continuous compounding formula is just another form of exponential function, its graph also has similar shape as the general form of the exponential function and its properties are also similar.

The graph of an decreasing exponential function of the form $A = a(e^r)^t$ with $a > 0$ and $0 < e^r < 1$ or $r < 0$, and the increasing exponential function with $a > 0$ and $e^r > 1$ or $r > 0$ are shown in the figure below



The domain in both lines is all real numbers while their range is all real numbers greater than zero, and the vertical intercept is $(0, a)$.

Therefore, we can conclude that although the three forms of exponential functions studied above look different as we can compare them below

$$y = ab^x \qquad A = P\left(1 + \frac{r}{n}\right)^{nt} \qquad A = a(e^r)^t$$

all of these equations are exponential functions, and this is what make them have same shape of graph and similar properties.