Exponents

Zero Identity: Whenever there is a nonzero base raised to the zero power, the value is ***always*** one.

General Form: $a^0 = 1 \text{ where } a \neq 0$

Fractional Exponents: For real numbers, roots can be written as fractional exponents. When given a fractional exponent, the numerator of the fraction is the power and the denominator is the root.

General Form: $a^{m/n} = \sqrt[n]{a^m} \text{ or } \sqrt[n]{a}^m \text{ where } n \geq 2$

Examples: $125^{2/3} = \sqrt[3]{125}^2 = 5^2 = 25$ $\qquad -8^{4/3} = \sqrt[3]{-8}^4 = (-2)^4 = 16$

Negative Exponents: If an expression has a ***negative exponent***, move ***only that expression*** to the other side of a fraction ***and*** make the exponent positive. [If it was in the numerator (top), move it to the denominator (bottom). If it was in the bottom, move it to the top.] (Coefficients must be combined as well.)

General Form: $a^{-n} = \frac{1}{a^n} \quad \text{or} \quad \frac{1}{a^{-n}} = \frac{a^n}{1}$

Example: $\frac{10x^{-2}z^5}{6y^{-4}} = \frac{5y^4z^5}{3x^2}$

Multiplication Rule: If multiple expressions with ***the same base*** are being ***multiplied***, rewrite the base and ***ADD*** the exponents. (Coefficients must be multiplied as well.)

General Form: $a^m \cdot a^n = a^{m+n}$

Examples: $x^2 \cdot x^3 = x^{2+3} = x^5$ $\qquad (2x^3)\cdot(5x^4) = 2\cdot 5 \cdot x^{3+4} = 10x^7$

Division Rule: If multiple expressions with ***the same base*** are being ***divided***, subtract the smaller exponent from the larger, and leave the same base raised to the result on the side of the fraction that had the larger exponent. (Coefficients must be divided/reduced as well.)

General Form: $\frac{a^m}{a^n} = \frac{a^{m-n}}{1} \quad \text{or} \quad \frac{a^m}{a^n} = \frac{1}{a^{n-m}}$

Examples: $\frac{4x^5}{2x^2} = \frac{4}{2}\cdot x^{5-2} = 2x^3$ $\qquad \frac{5y^4}{10y^6} = \frac{5}{10}\cdot\frac{1}{y^{6-4}} = \frac{1}{2y^2}$

Exponents raised to another exponent: When you have an exponent raised to another exponent, you rewrite the base and ***MULTIPLY*** the exponents.

General Form: $(a^m)^n = a^{m\cdot n}$

Examples: $(x^3)^4 = x^{3\cdot 4} = x^{12}$ $\qquad (-2m^4n^2)^3 = (-2^{\ 3})m^{4\cdot 3}n^{2\cdot 3} = -8m^{1\ 2}n^6$

Combine the rules together: $\frac{3w^0x^3y^{-1}z^9}{9x^{-2}y^5(z^2)^3} = \frac{3\cdot 1\cdot x^3x^2z^9}{9y^5y^1z^6} = \frac{3x^{3+2}z^{9-6}}{9y^{5+1}} = \frac{x^5z^3}{3y^6}$

(w^0 becomes 1; move negative exponents; multiply exponents raised to exponents; add those that are multiplied; subtract those that are divided; reduce coefficients)