Factoring Polynomials

		EXAMPLES
	STEP 1 : Greatest Common Factor	
GCF	Factor out the greatest common factor.	$6x^3 + 12x^2y = 6x^2 (x + 2y)$ $5x - 5 = 5 (x - 1)$ $7x^2 + 2y^2 = 1 (7x^2 + 2y^2)$ $2x (x - 3) - (x - 3) = (x - 3) (2x - 1)$
	STEP 2 : Count Terms	
Two Terms (Binomials)	1) Difference of two squares. $a^2 - b^2 = (a + b) (a - b)$ or $(a - b) (a + b)$	$4 - 9x^2 = (2)^2 - (3x)^2$ $=(2 + 3x) (2 - 3x)$
		$x^4 - 1 = (x^2)^2 - (1)^2$ $= (x^2 + 1) (x^2 - 1)$ $= (x^2 + 1) (x - 1) (x + 1)$
	2) Difference of two cubes. $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$	$8x^3 - 27 = (2x)^3 - (3)^3$ $= (2x - 3) ((2x)^2 + (2x)(3) + (3)^2)$ $= (2x - 3) (4x^2 + 6x + 9)$
		$y^3 - 1 = (y)^3 - (1)^3$ $= (y - 1) (y^2 + y + 1)$
	3) Sum of two cubes $a^3 + b^3 = (a + b) (a^2 - ab + b^2)$	$8x^3 + 27 = (2x)^3 + (3)^3$ $= (2x + 3) ((2x)^2 - (2x)(3) + (3)^2)$ $= (2x + 3) (4x^2 - 6x + 9)$
		$y^3 + 1 = (y)^3 + (1)^3$ $= (y + 1) (y^2 - y + 1)$
Three Terms (Trinomials)	1) Leading coefficient is 1. $x^2 + bx + c$ Find two integers whose product = **c** and whose sum = **b**	$x^2 - 5x - 24$ Find two integers whose product = -24 and whose sum = -5. $-8 \cdot 3 = -24$ and $-8 + 3 = -5$ so, $x^2 - 5x - 24 = (x - 8) (x + 3)$
		$x^2 + x - 12$ Find two integers whose product = -12 and whose sum = 1. $4 \cdot (-3) = -12$ and $4 + (-3) = 1$, so, $x^2 + x - 12 = (x + 4) (x - 3)$.
		$x^2 - 5x + 6$ Find two integers whose product = 6 and whose sum = -5. $(-3) \cdot (-2) = 6$ and $(-3) + (-2) = -5$, so, $x^2 - 5x + 6 = (x - 3) (x - 2)$.

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<td rowspan="2"></td>
<td rowspan="2">2) Leading coefficient is -1.

$-x^2 + bx + c$ factor out -1 first.
$-1 (x^2 - bx - c)$ then factor $(x^2 - bx - c)$
using step 1 above.</td>
<td>$-x^2 + 5x + 24$
$= -(x^2 - 5x - 24)$
$= -(x - 8) (x + 3)$</td>
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<tr>
<td>$-x^2 + 5x - 6$
$= -(x^2 - 5x + 6)$
$= -(x - 3) (x - 2)$</td>
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<td rowspan="3"></td>
<td>3) Leading coefficient is not 1 or -1.
$ax^2 + bx + c$, $a \neq 1$, -1

i) you may use your calculator to check if this is factorable. Find the <u>zeros.</u>
If the zeros are integers or rational numbers, then the given trinomial is factorable. Otherwise, the trinomial is <u>prime.</u></td>
<td>$18x^2 + 3x - 10$

Zeros (by calculator) are 2/3 and -5/6 , so this polynomial is factorable. The factors are (3x - 2) and (6x + 5). So,
$18x^2 + 3x - 10 = (3x - 2) (6x + 5)$.

On the other hand,
$18x^2 + 3x + 10$ has imaginary zeros and,
$5x^2 - 10x + 3$ has irrational zeros so both are prime.</td>
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<td>ii) <u>ac</u><u> - method</u>: $ax^2 + bx + c$
Find the product of the coefficients a and c. Then find two integers whose product = ac and whose sum = b.
Rewrite the middle term replacing b by the sum of the two integers. Factor the resulting four-term polynomial by grouping.

(See section below
on factoring four-term polynomials.)</td>
<td>$18x^2 + 3x - 10$

ac = (18) (-10) = -180
Find two integers whose product = -180 and whose sum = 3.
(15) (-12) = -180
15 + (-12) = 3
so, $18x^2 + 3x - 10$
$= 18x^2 + 15x - 12x - 10$
$= 3x (6x + 5) - 2 (6x + 5)$ by grouping
$= (6x + 5) (3x - 2)$

You may organize the terms in a box and find the greatest common factor of each row and each column, factoring out leading minus signs.

I I
I $18x^2$ I $+15x$
I $-12x$ I -10

I 6x I +5
3x I $18x^2$ I $+15x$
-2 I $-12x$ I -10

so, $18x^2 + 3x - 10 = (3x - 2) (6x + 5)$</td>
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iii) Trial and error method $ax^2 + bx + c$

Write possible factorizations of **a** and of **c**.

Find the sum of the outer and inner products. We need this sum to equal **b**x.

If **b** and **c** are both positive, the factors have only positive coefficients.

$12x^2 + 43x + 35$

$12 = 12\cdot 1 = 6\cdot 2 = 4\cdot 3$ $\qquad$ $35 = 35\cdot 1 = 5\cdot 7$

$(12x + 35)(x + 1)$	$(12x + 1)(x + 35)$
$35x$	x
$12x$	$420x$
$35x + 12x = 47x \neq 43x$	$x + 420x = 421x \neq 43x$
$(12x + 5)(x + 7)$	$(12x + 7)(x + 5)$
$5x$	$7x$
$84x$	$60x$
$5x + 84x = 89x \neq 43x$	$7x + 60x = 67x \neq 43x$
$(6x + 5)(2x + 7)$	$(6x + 7)(2x + 5)$
$10x$	$14x$
$42x$	$30x$
$10x + 42x = 52x \neq 43x$	$14x + 30x = 44x \neq 43x$
$(4x + 7)(3x + 5)$	$(4x + 5)(3x + 7)$
$21x$	$15x$
$20x$	$28x$
$21x + 20x = 41x \neq 43x$	$15x + 28x = 43x$

so, $12x^2 + 43x + 35 = (4x + 5)(3x + 7)$

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If **b** is positive and **c** is negative, one of the factors of **c** must be negative and the sum of the outside and inside products must be positive.

$18x^2 + 3x - 10$

$18 = 18\cdot 1 = 9\cdot 2 = 6\cdot 3$ $\qquad$ $-10 = -10\cdot 1 = 10\cdot(-1) = -5\cdot 2 = 5\cdot(-2)$

$18\cdot 1$ and $-10\cdot 1$ or $10\cdot(-1)$ most likely will not work because 3 will not be a sum.

$(9x - 5)(2x + 2)$	$(9x - 2)(2x + 5)$
$-10x$	$-4x$
$18x$	$45x$
$-10x + 18x = 8x \neq 3x$	$-4x + 45x = 41x \neq 3x$

$(6x + 5)(3x - 2)$

$15x$

$-12x$

$15x + -12x = 3x$

so, $18x^2 + 3x - 10 = (6x + 5)(3x - 2)$

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<td>If b and c are both negative, one of the factors of c must be negative and the sum of the outside and inside products must be negative.</td>
<td>$12x^2 - 13x - 35$

$12 = 12\cdot1$ $-35 = -35\cdot1 = 35\cdot(-1)$
$= 6\cdot2$ $= -5\cdot7 = 5\cdot(-7)$
$= 4\cdot3$

$(6x - 5)(2x + 7)$
$-10x$
$42x$
$-10x + 42x = 32x \neq -13x$

$(4x - 5)(3x + 7)$ $(4x + 5)(3x - 7)$
$-15x$ $15x$
$28x$ $-28x$
$-15x + 28x = 13x \neq -13x$ $15x - 28x = -13x$

so, $12x^2 - 13x - 35 = (4x + 5)(3x - 7)$</td>
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<td rowspan="2">Four Terms</td>
<td rowspan="2">If there are four terms, consider factoring by grouping the terms into two groups of two terms each. Factor the GCF from each group. If there is then a common binomial factor, factor it out.</td>
<td>$18x^2 + 3x - 10$

$18x^2 + 15x - 12x - 10$

$= 3x(6x + 5) - 2(6x + 5)$
$= (6x + 5)(3x - 2)$

You may organize the terms in a box and find the greatest common factor of each row and each column, factoring out leading minus signs.

| |
| $18x^2$ | $+15x$
| $-12x$ | -10

| 6x | +5
3x | $18x^2$ | $+15x$
-2 | $-12x$ | -10

so, $18x^2 + 3x - 10 = (3x - 2)(6x + 5)$</td>
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<td>$12x^2 - 28x + 15x - 35$

$= 4x(3x - 7) + 5(3x - 7)$
$= (3x - 7)(4x + 5)$

OR

| 3x | -7
4x | $12x^2$ | $-28x$
+5 | $+15x$ | -35</td>
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<tr><td></td><td></td><td>$3x^3 + 7x^2 - 15x - 35$

$= x^2 (3x + 7) - 5(3x + 7)$
$= (3x + 7) (x^2 - 5)$

OR

$\begin{array}{c|c|c} & \mathbf{3x} & \mathbf{-7} \\ \hline \mathbf{x^2} & 3x^3 & +7x^2 \\ \hline \mathbf{-5} & -15x & -35 \end{array}$</td></tr>
<tr><td></td><td colspan="2">STEP 3 : Factor Completely</td></tr>
<tr><td></td><td>Examine each non-monomial factor resulting from the above steps and check if it is further factorable

Repeat this step until each non-monomial factor is <u>prime.</u></td><td>$x^4 - 81$ (Difference of 2 squares)
$= (x^2 + 9) (x^2 - 9)$ ($x^2 - 9$ is also Difference of 2 squares)
$= (x^2 + 9) (x + 3) (x - 3)$
<hr>$5x^4 - 405$
$= 5 (x^4 - 81)$
$= 5 (x^2 + 9) (x^2 - 9)$
$= 5 (x^2 + 9) (x - 3) (x + 3)$
<hr>$x^{12} - y^6$
$= (x^6)^2 - (y^3)^2$
$= (x^6 - y^3) (x^6 + y^3)$
$= ((x^2)^3 - y^3) ((x^2)^3 + y^3)$
$= (x^2 - y) (x^4 + x^2y + y^2) (x^2 + y) (x^4 - x^2y + y^2)$
<hr>$48x^4 + 108x^3 - 30x^2$
$= 6x^2 (8x^2 + 18x - 5)$
$= 6x^2 (8x^2 + 20x - 2x - 5)$
$= 6x^2 (2x + 5) (4x - 1)$

where

$\begin{array}{c|c|c} & \mathbf{2x} & \mathbf{+5} \\ \hline \mathbf{4x} & 8x^2 & +20x \\ \hline \mathbf{-1} & -2x & -5 \end{array}$</td></tr>
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Note: Calculators with symbolic manipulation features can factor polynomials directly. Regardless of the method used to factor polynomials, the importance of factors is their use in solving problems.