Fractions

What is a fraction: A rational number is the ratio of two integers and is denoted by a fraction. A "fraction" consists of two parts: the top number is called the *numerator* and the bottom number is called the *denominator*. A fraction may also be written in the form of a decimal, but not all decimal numbers may be written as a fraction (these numbers are called "*irrational*").

$$\text{fraction} = \frac{\text{numerator}}{\text{denominator}}$$

$\pi = 3.14159....$ is irrational because it cannot be written as a fraction.

Mixed numbers → "Improper" Fractions: Mixed numbers are the sum of a whole number and a fraction. To turn a mixed number into a single fraction, multiply the whole number by the denominator and add that value to the numerator of the fraction to calculate the numerator. The denominator of the single fraction is the same as the denominator of the original fraction.

Mixed numbers: $3\frac{1}{4} = 3 + \frac{1}{4}$ $\qquad -2\frac{2}{3} = -\left(2 + \frac{2}{3}\right)$ $\qquad$ *Examples*: $4\frac{1}{2} = \frac{4 \cdot 2 + 1}{2} = \frac{9}{2}$ $\qquad 3\frac{5}{7} = \frac{3 \cdot 7 + 5}{7} = \frac{26}{7}$

One Identity: Any nonzero number divided by itself is equal to 1.

Examples: $\frac{2}{2} = 1$ $\qquad \frac{15}{15} = 1$ $\qquad \frac{-8}{-8} = 1$ $\qquad \frac{.5}{.5} = 1$ $\qquad \frac{\pi}{\pi} = 1$

Reducing Fractions: In order to reduce a fraction to its smallest/simplest form, factor the numerator and the denominator into their smaller/smallest parts, and then "cancel" out any numbers common to the both the top and bottom (this is actually a use of the *One Identity*).

Examples: $\frac{6}{14} = \frac{2 \cdot 3}{2 \cdot 7} = 1 \cdot \frac{3}{7} = \frac{3}{7}$ or $\frac{6}{14} = \frac{2 \cdot 3}{2 \cdot 7} = \frac{3}{7}$ $\qquad \frac{60}{66} = \frac{2 \cdot 2 \cdot 3 \cdot 5}{2 \cdot 3 \cdot 11} = 1 \cdot 1 \cdot \frac{2 \cdot 5}{11} = \frac{10}{11}$ or $\frac{60}{66} = \frac{2 \cdot 2 \cdot 3 \cdot 5}{2 \cdot 3 \cdot 11} = \frac{2 \cdot 5}{11} = \frac{10}{11}$

Multiplication: When multiplying fractions, multiply all of the numerators together to find the numerator of the answer, and then multiply all of the denominators together to find the denominator of the answer. If your answer must be in its reduced form, then it may be useful to reduce as you go.

Examples: $\frac{1}{2} \cdot \frac{5}{3} = \frac{1 \cdot 5}{2 \cdot 3} = \frac{5}{6}$ $\qquad\qquad \frac{3}{4} \cdot \frac{-12}{5} = \frac{-3 \cdot 12}{4 \cdot 5} = \frac{-3 \cdot 3 \cdot 4}{4 \cdot 5} = -\frac{9}{5}$

Division: When two fractions are divided, "flip" the second fraction, and then multiply. This is called multiplying by the *reciprocal* of the second fraction.

Example: $\frac{2}{3} \div \frac{5}{2} = \frac{2}{3} \cdot \frac{2}{5} = \frac{2 \cdot 2}{3 \cdot 5} = \frac{4}{15}$

Addition and Subtraction: If fractions have the same denominator, you simply add/subtract the numerators and put the result over that denominator. If fractions have different denominators, multiply each fraction by a fraction equal to one (a number divided by itself) in order to end up with the same denominator for all of the fractions. Then add or subtract the resulting fractions.

Examples: $\frac{1}{5} + \frac{2}{5} = \frac{1 + 2}{5} = \frac{3}{5}$ $\qquad\qquad \frac{1}{2} + \frac{1}{3} = \frac{1}{2} \cdot \frac{3}{3} + \frac{1}{3} \cdot \frac{2}{2} = \frac{1 \cdot 3}{2 \cdot 3} + \frac{1 \cdot 2}{3 \cdot 2} = \frac{3}{6} + \frac{2}{6} = \frac{3 + 2}{6} = \frac{5}{6}$

Combine the rules together:

$$\left(3\frac{1}{2} \div 1\frac{1}{3}\right) - \frac{5}{2} = \left(\frac{3 \cdot 2 + 1}{2} \div \frac{1 \cdot 3 + 1}{3}\right) - \frac{5}{2} = \left(\frac{7}{2} \div \frac{4}{3}\right) - \frac{5}{2} = \left(\frac{7}{2} \cdot \frac{3}{4}\right) - \frac{5}{2} = \frac{21}{8} - \frac{5}{2} = \frac{21}{8} - \frac{5}{2} \cdot \frac{4}{4} = \frac{21}{8} - \frac{20}{8} = \frac{21 - 20}{8} = \frac{1}{8}$$

Convert mixed numbers; "flip and multiply"; get common denominators; subtract the numerator.