Functions

Types of Functions:

Polynomial: ex. $F(x) = 4x^5 + 3x^3 + 2x + 10$

Linear: The highest degree of the expression is one. ex. $f(x) = 3x + 5$

Quadratic: The highest degree of the expression is two. ex. $f(x) = 3x^2 + 5x$

Cubic: The highest degree of the expression is three. ex. $f(x) = 5x^3 + 7x + 8$

Exponential: An exponential function has the form $f(x) = a^n$, where $a>0$, $a\neq 1$ and the constant real number a is called the base.

Logarithmic: The common logarithm, log x, has no base indicated and the understood base is always 10. The natural logarithm, ln x, has no base indicated, is written Ln instead of Log, and the understood base is always e.

Rational: $f(x) = \frac{P(x)}{Q(x)}$ where P(x) and Q(x) are polynomials which are relatively prime (lowest terms). Q(x) has a degree greater than zero, and $Q(x) \neq 0$. ex $f(x) = \frac{5x + 3}{7x - 10}$

Radical: $f(x) = \sqrt{P(x)}$ where P(x) is a polynomial ex. $f(x) = \sqrt{4x + 5}$

Function tests:

Horizontal Line Test: Indicates a one-to-one function if no horizontal intersects more than one point.

Vertical Line Test: Indicates a relation is also a function if no vertical line intersects the graph of the relation at more than one point.

Domain:

Rational Functions: The denominator cannot be zero

$\frac{x + 3}{(x+2)(x-3)}$ *Domain:* All real numbers except -2 or 3

$(-\infty,-2)\cup(-2,3)\cup(3,\infty)$ Interval Notation

Radical Functions: What's under the radical must be greater than or equal to zero

ex $\sqrt{x+3}$ $x+3 \geq 0$

-3 -3 *Domain:* All real numbers greater than or equal to –3

$x \geq -3$ $[-3,\infty)$ Interval Notation

Range: The y-values that your graph includes.

Evaluate:

When evaluating a function, substitute the value that you want to evaluate the function at into all occurrences of the variables and solve.

ex. find the value of $3x^2 + 4x + 5$ when $x = 3$

$3(3)^2 + 4(3) + 5 = 27 + 12 + 5 = 44$

find f(2) if $f(x) = 2x + 10$

$2(2) + 10 = 14$

Inverse:

When finding the inverse of a function, replace the x-variable with y and y-variable with x, and solve for y and replace y with $f^{-1}(x)$.

ex. $y = 5x + 6 \Rightarrow x = 5y + 6 \Rightarrow \frac{x-6}{5} = \frac{5y}{5} \Rightarrow f^{-1}(x) = \frac{x-6}{5}$

(-6 -6)

Composition Of Functions: $f[g(x)]$ or $f \circ g$ To solve a composite function substitute the second function into the first wherever there is a variable and simplify.

ex. Find $f[g(x)]$ if $f(x) = 3x - 8$ and $g(x) = 5x + 6$

$$f[g(x)] = 3(5x + 6) - 8$$
$$= 15x + 18 - 8$$
$$f[g(x)] = 15x + 10$$

Asymptotes: Horizontal Asymptotes: A horizontal line which the graph of the function approaches as $x \to \pm \infty$. The graph of the function may cross a horizontal asymptote.

Finding Horizontal Asymptotes:

- Highest exponent in numerator is greater than the highest exponent in denominator then there is not a horizontal asymptote.
  ex. $\frac{5x^3 + 4x^2}{3x + 10}$ = No horizontal asymptote
- Highest exponent in denominator is greater than the highest exponent in numerator then the horizontal asymptote is always $y = 0$. ex. $\frac{4x^2 + 9x + 10}{3x^3}$ $y = 0$ is horizontal asymptote
- Highest exponent in numerator and highest exponent in denominator are equal, then the horizontal asymptote is the ratio of their coefficients. ex. $\frac{5x^2 + 3x}{7x^2 + 4x}$ $y = \frac{5}{7}$ is horizontal asymptote

Vertical Asymptotes: A vertical line which the graph of the function approaches as $y \to \pm \infty$. The graph of the function will never cross a vertical asymptote.

Finding Vertical Asymptotes:

- Set the denominator equal to zero (if there is not a denominator, then there is no vertical asymptote).
- Solve that equation for "x".
- Once you have what "x" equals, that is your vertical asymptote.

Symmetry: Graphs are symmetric with respect to a line if, when folded along the drawn line, the two parts of the graph then land upon each other.

Tests for Symmetry:

- If replacing "y" with "-y" results in an equivalent equation, then the graphs are symmetric to the x-axis.
- If replacing "x" with "-x" results in an equivalent equation, then the graphs are symmetric to the y-axis.
- If replacing both "x" with "-x" and "y" with "-y" results in an equivalent equation, then the graphs are symmetric to the origin.