

Hypothesis Testing about a Population Mean

1. State the null (H_0) and alternative (H_a) hypotheses in plain English
2. State the null and alternative hypotheses using the correct statistical measure (the value of “a” is the hypothesized mean given in the problem)
 - There are three possibilities:
 - Upper-tailed test – Testing the claim “greater than”

$$H_0 : \mu \leq a$$

$$H_a : \mu > a$$
 - Lower-tailed test – Testing the claim “less than”

$$H_0 : \mu \geq a$$

$$H_a : \mu < a$$
 - Two-tailed test – Testing the claim “not equal to”

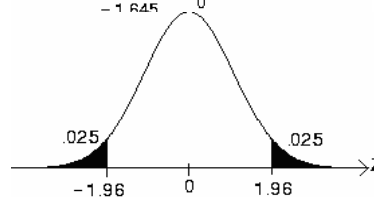
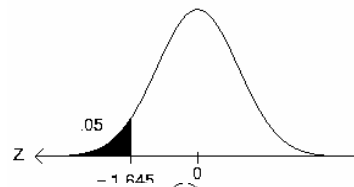
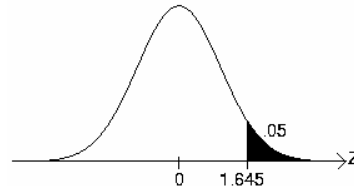
$$H_0 : \mu = a$$

$$H_a : \mu \neq a$$
3. Specify the α -level of the test
 - The level of the test determines how rare an event must be in order to reject the null hypothesis
 - This is usually given to you in the statement of the problem
 - Typical values of α are .10, .05, and .01
4. Determine your test statistic: either use a z-test or t-test
 - There are three key questions to ask:
 - Is the variance of the population known?
 - Is the distribution of the population normal (or approx. normal)?
 - Is the sample size large?
 1. When the variance is known **and** either the distribution is normal or sample size is large, use a z-test statistic
 2. When the variance is unknown **and** a sample size less than 30, use a t-test statistic as long as the population is normal (or approx. normal)
5. Determine the critical value of the test statistic and the rejection region
 - This depends on the type of test you are doing (upper, lower or two tailed), the α -level of the test, and the distribution of the test statistic

• **FOR EXAMPLE:**

$\alpha=.05$ with a z-test statistic (normal distribution)

- Upper Tailed Test - Reject the null hypothesis if the sample test statistic is greater than 1.645
- Lower Tailed Test - Reject the null hypothesis if the sample test statistic is less than -1.645
- Two Tailed Test - Reject the null hypothesis if the sample test statistic is greater than 1.96 or less than -1.96



6. Compute sample test statistic
 - Use the z or t calculation depending on your choice from step 4
 - \bar{x} - sample mean
 - μ - hypothesized mean (“a” from step 2)
 - σ - population standard deviation
 - s - sample standard deviation
 - n - sample size

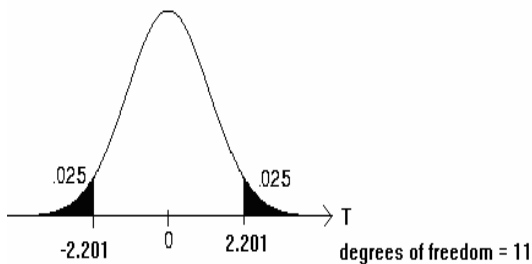
$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \qquad t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

7. Make a decision based on your sample test statistic
 - If the value you calculated in step 6 is in the rejection region, reject the null hypothesis in favor of the alternative
 - If the value you calculated in step 6 is NOT in the rejection region, do not reject the null hypothesis
8. State the conclusion in terms of the original question

Example:

Researchers studying the effects of diet on growth would like to know if a vegetarian diet affects the height of a child. The researchers randomly selected 12 vegetarian children that are six years old. The average height of the children is 42.5 inches with a standard deviation of 3.8 inches. The average height for all six year old children is 45.75 inches. Conduct a hypothesis test to determine whether there is overwhelming evidence at $\alpha=.05$ that six year old vegetarian children are not the same height as other six year old children. Assume that the heights of six year old vegetarian children are approximately normally distributed.

1. The null hypothesis – Six year old vegetarian children **ARE** the same height as other six year old children.
The alternative hypothesis – Six year old vegetarian children **ARE NOT** the same height as other six year old children.
2. $H_0 : \mu = 45.75$
2. $H_a : \mu \neq 45.75$
3. $\alpha=.05$ (as stated in the problem)
4. Since we only selected 12 vegetarian children, we have a small sample size which means we use a t -test statistic.



5. Since we are conducting a two-tailed test, our rejection region would have t -critical values of 2.201 and -2.201. These are gotten by looking at the t -table with $12 - 1=11$ degrees of freedom and $\alpha=.025$.

6. Our test statistic is:

$$t = \frac{42.5 - 45.75}{\frac{3.8}{\sqrt{12}}} = \frac{-3.25}{1.097} = -2.963$$

7. Since $-2.963 < -2.201$ we reject H_0 in favor of the

alternative hypothesis.

8. There is overwhelming evidence at the $\alpha=.05$ level that vegetarian six year olds are not the same height as regular six year olds. (Note: We can not conclude they are shorter because our initial test was not designed to answer that question. You would have to do another hypothesis test to see if vegetarian six year old children were shorter than other six year old children)