Hypothesis Testing about a Population Proportion

1. State the null (H_o) and alternative (H_α) hypotheses in plain English
2. State the null and alternative hypotheses using the correct statistical measure (the value of "a" is the hypothesized proportion given in the problem)
    - There are three possibilities:
        - Upper-tailed test – Testing the claim of "greater than"

          $H_o : p \le a$

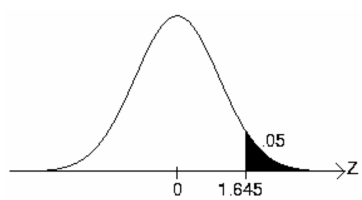
          $H_\alpha : p > a$
        - Lower-tailed test – Testing the claim of "less than"

          $H_o : p \ge a$

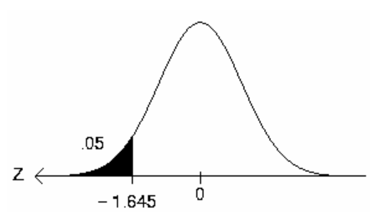
          $H_\alpha : p < a$
        - Two-tailed test – Testing the claim of "not equal to"

          $H_o : p = a$

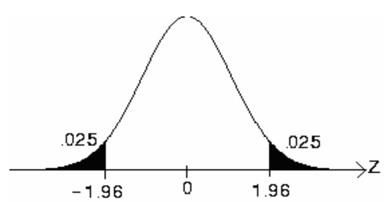
          $H_\alpha : p \ne a$
3. Specify the α-level of the test
    - The level of the test determines how rare an event must be in order to reject the null hypothesis
    - This is typically given to you in the statement of the problem
    - Typical values of α are .10, .05, and .01
4. Determine your test statistic:
    - If np ≥ 5 and n(1-p) ≥ 5, use a *z*-test statistic
5. Determine the critical value of the test statistic and the rejection region
    - This depends on the type of test you are doing (upper, lower or two tailed), the α-level of the test, and the distribution of the test statistic
    - **FOR EXAMPLE:** α=.05 with a *z*-test statistic (normal distribution)
        - Upper Tailed Test - Reject the null hypothesis if the sample test statistic is greater than 1.645



        - Lower Tailed Test - Reject the null hypothesis if the sample test statistic is less than -1.645



        - Two Tailed Test - Reject the null hypothesis if the sample test statistic is greater than 1.96 or less than -1.96

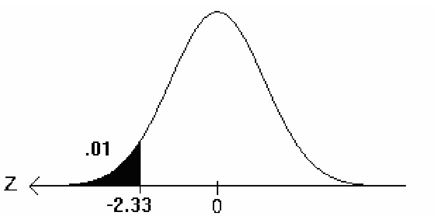


6. Compute sample test statistic
    - $\hat{p}$ -sample proportion
    - p -hypothesized proportion ("a" from step number 2)
    - n - sample size
        - $z = \dfrac{\hat{p} - p}{\sqrt{\dfrac{p(1-p)}{n}}}$

7. Make a decision based on your sample test statistic
   - If the value you calculated in step 6 is in the rejection region, reject the null hypothesis in favor of the alternative
   - If the value you calculated in step 6 is NOT in the rejection region, do not reject the null hypothesis
8. State the conclusion in terms of the original question

Example:
Ships arriving in U.S. ports are inspected by Customs officials for contaminated cargo. Assume, for a certain port, 20% of the ships arriving in the previous year contained cargo that was contaminated. A random selection of 50 ships in the current year included 5 that had contaminated cargo. Does the data suggest that the proportion of ships arriving in the port with contaminated cargoes has decreased in the current year? Use α=.01.

1. The null hypothesis – The proportion of ships arriving into the port this year with contaminated cargo is at least .20
   The alternative hypothesis – The proportion of ships arriving into the port this year with contaminated cargo is less than .20
2. $H_o : p \geq .2$
   $H_\alpha : p < .2$
3. α=.01 (as stated in the problem)
4. Since 50(.2) = 10 and 50(.8) = 40 which are both bigger than 5 we can use a *z*-test statistic



5. Since we are conducting a lower-tailed test our rejection region will have a *z*-critical value of -2.33
6. Our test statistic is:

$$z = \frac{\frac{5}{50} - .2}{\sqrt{\frac{.2(1-.2)}{50}}} = \frac{.1-.2}{.0566} = \frac{-.1}{.0566} = -1.767$$

7. Since -2.33 < -1.767 we fail to reject H_o
8. There is not overwhelming evidence at the α=.01 level that the proportion of ships arriving to the port this year with contaminated cargo has decreased since last year.