Inequalities

Properties

Addition: (Similar rules hold for subtraction.)

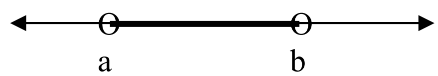
$a<b$ if and only if $a+c<b+c$
$a\le b$ if and only if $a+c\le b+c$
$a>b$ if and only if $a+c>b+c$
$a\ge b$ if and only if $a+c\ge b+c$

Multiplication: (Similar rules hold for division.)

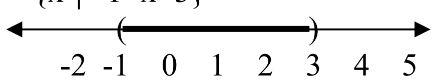
If $c>0$ and $a<b$ then $ac<bc$
If $c>0$ and $a>b$ then $ac>bc$
If $c<0$ and $a<b$ then $ac>bc$
If $c<0$ and $a>b$ then $ac<bc$

Interval Notation and Number Line Graphs

(a,b) = the set of all numbers between a and b
= $\{x \mid a<x<b\}$

or


e.g., $(-1,3)$ = the set of all numbers between –1 and 4
= $\{x \mid -1<x<3\}$



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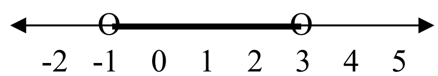
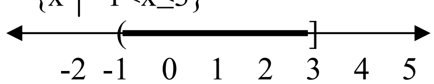


$(a,b]$ = the set of all numbers greater than a and less than or equal to b
= $\{x \mid a<x\le b\}$

or


e.g., $(-1,3]$ = the set of all numbers greater than –1 and less than or equal to 4
= $\{x \mid -1<x\le 3\}$



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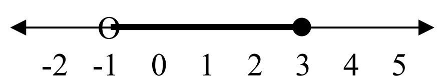
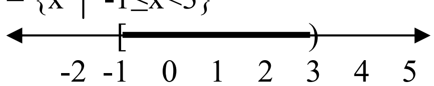


$[a,b)$ = the set of all numbers greater than or equal to a and less than b
= $\{x \mid a\le x<b\}$

or


e.g., $[-1,3)$ = the set of all numbers between greater than or equal to –1 and less than 4
= $\{x \mid -1\le x<3\}$



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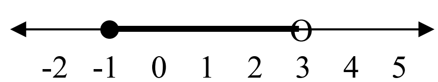
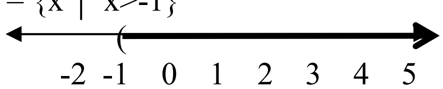


$(a,{}^{+}\infty)$ = the set of all numbers greater than a
= $\{x \mid x>a\}$

or


e.g., $(-1,{}^{+}\infty)$ = the set of all numbers greater than -1
= $\{x \mid x>-1\}$



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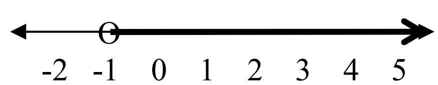
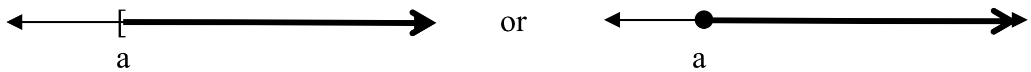
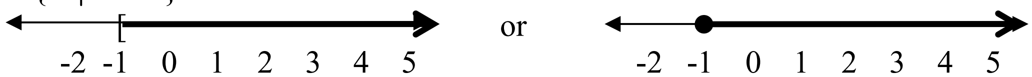


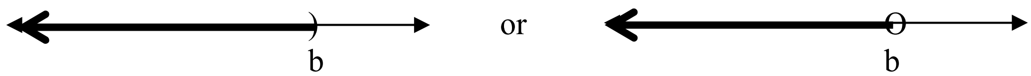
$[a,{}^{+}\infty)$ = the set of all numbers greater than or equal to a
= $\{x \mid x \geq a\}$



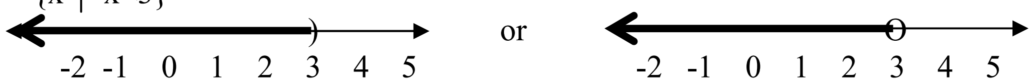
e.g., $[-1, {}^{+}\infty)$ = the set of all numbers greater than or equal to -1
= $\{x \mid x \geq -1\}$



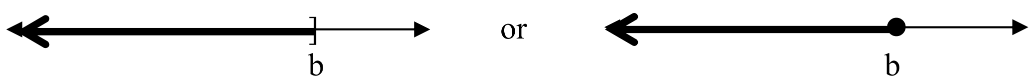
$({}^{-}\infty,b)$ = the set of all numbers less than b
= $\{x \mid x<b\}$



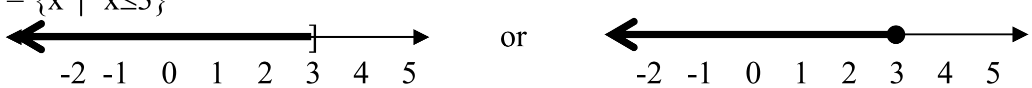
e.g., $({}^{-}\infty.3)$ = the set of all numbers less than 3
= $\{x \mid x<3\}$



$({}^{-}\infty,b]$ = the set of all numbers less than or equal to b
= $\{x \mid x \leq b\}$



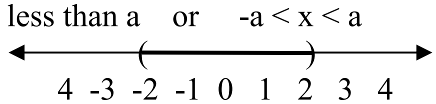
e.g., $({}^{-}\infty.3]$ = the set of all numbers less than or equal to 3
= $\{x \mid x \leq 3\}$



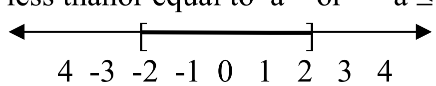
$({}^{-}\infty,{}^{+}\infty)$= the set of all real numbers

Absolute Value Inequalities (a>0)

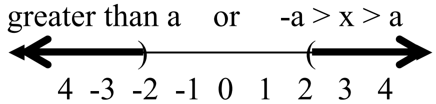
$|x| < a$ means that x is a number whose distance from 0 is less than a or $-a < x < a$
e.g., $|x| < 2$ means $-2 < x < 2$



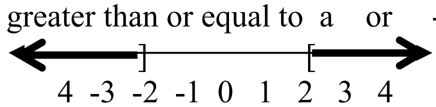
$|x| \leq a$ means that x is a number whose distance from 0 is less thanor equal to a or $-a \leq x \leq a$
e.g., $|x| \leq 2$ means $-2 \leq x \leq 2$



$|x| > a$ means that x is a number whose distance from 0 is greater than a or $-a > x > a$
e.g., $|x| > 2$ means $-2 > x > 2$



$|x| \geq a$ means that x is a number whose distance from 0 is greater than or equal to a or $-a \geq x \geq a$
e.g., $|x| \geq 2$ means $-2 \geq x \geq 2$



To solve $|cx + d| < a$ for x

(1) Rewrite the inequality as $-a < cx + d < a$

(2) Use the properties of inequalities to isolate x in the middle

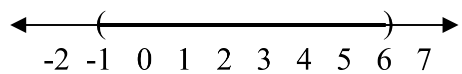
e.g. $|2x - 5| < 7$ Rewrite the inequality.

$-7 < 2x - 5 < 7$ Add 5 to all three parts.

$-2 < 2x < 12$ Divide all three parts by 2 which is > 0

$-1 < x < 6$

Solution: (-1,6)



The above steps will work for $|cx + d| \le a$ as well.

To solve $|cx + d| > a$ for x

(1) Rewrite the inequality as $cx + d < -a$ or $cx + d > a$

(2) Solve each inequality in step (1).

(3) The solution is the <u>union</u> of the two solution sets.

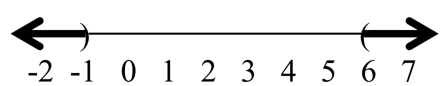
e.g. $|2x - 5| > 7$ Rewrite the inequality.

$2x - 5 < -7$ or $2x - 5 > 7$ Add 5 to all three parts.

$2x < -2$ or $2x > 12$ Divide all three parts by 2 which is > 0

$x < -1$ or $x > 6$

Solution $(^{-}\infty,-1) \cup (6,^{+}\infty)$



The above steps will work for $|cx + d| \ge a$ as well.

In a situation where the variable has a negative multiplier $|-cx| < a$

(1) Rewrite the inequality as $-a < -cx < a$

(2) When dividing the negative multiplier, the direction of all of the signs must change as well

(< goes to >, > goes to <, ≥ goes to ≤, ≤ goes to ≥)

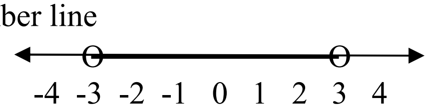
e.g. $|-2x| < 6$ Rewrite the inequality

$-6 < -2x < 6$ Divide both sides by -2 and switch the direction of the signs

$3 > x > -3$ Flip the entire equation around so that is will coincide with the number line

$-3 < x < 3$

Solution (-3,3)



For all situations when a negative multiplier is involved, the direction of all of the signs must change when the multiplier is divided.