Interest/Time Value of Money (TVM)

Simple Interest: $A = P + P \cdot r \cdot t$

Continuous Interest: $A = P \cdot e^{r \cdot t}$

Annual Interest: $A = P(1+r)^t$

Annual % Yield: $APY = \left(1 + \frac{APR}{n}\right)^n - 1$

Compound Interest: $A = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$

Savings Plan Formula: $A = PMT \cdot \frac{\left[(1 + r/n)^{n \cdot t} - 1\right]}{r/n}$

Payment Formula: $PMT = \frac{P \cdot r/n}{\left[1 - (1 + r/n)^{-n \cdot t}\right]}$

TVM (Time Value of Money)

On TI-Plus Calculators → **APPS** : Finance: TVM Solver; On TI-83 Calculators → **2ND** : x^{-1} : TVM Solver

N=	Number of total payments. (n*t)
I%=	Annual Percentage Rate as a %, not a decimal
PV=	Present Value: Starting balance of a loan, or savings account
PMT=	Recurring payment
FV=	Future value/accrued amount. (0 for loans)
P/Y=	Payments per year (most often 12 for monthly payments)
C/Y=	Compounding cycles per year (n; often matches P/Y)
PMT:END BEGIN	Time when payments are made (almost always END unless specified)

IMPORTANT: If the money is coming to you, the value will be positive. If the money is leaving your hands, it is negative.

TO SOLVE FOR A VALUE: Move curser to whichever item you want to solve for and press ALPHA and then ENTER

Examples:

You put $1000 in a savings account that has an annual interest rate of 6% with annual compounding. Without any additional payments, what will the balance be after 10 years?

$A = 1000(1+.06)^{10} = \$1790.84$

N= 10
I%= 6
PV= -1000
PMT= 0
■FV= 1790.847697
P/Y= 1
C/Y= 1
PMT:END BEGIN

How much did you contribute?

$\$1000$

How much interest did you earn?

$\$1790.84 - 1000 = \$790.84$

If instead you decide to initially deposit $1000 in the account and make monthly payments of $20 compounded monthly, what will the balance be?

$A_{Total} = A_{\$1000} + A_{\$20\,pmt/mo.}$

$A_{\$1000} = \$1000 \cdot \frac{\left[(1+.06/12)^{12 \cdot 10} - 1\right]}{.06/12} = \$1819.40$

$A_{\$20\,pmt/mo} = 20 \cdot \frac{\left[(1+.06/12)^{12 \cdot 10} - 1\right]}{.06/12} = \$3277.58$

$A_{Total} = \$1819.40 + \$3277.59 = \$5096.98$

N= 120
I%= 6
PV= -1000
PMT= -20
■FV= 5096.98367
P/Y= 12
C/Y= 12
PMT:END BEGIN

How much did you contribute?

$\$1000 + \$20 \cdot 12 \cdot 10 = \$3400$

How much interest did you earn?

$\$5096.98 - \$3400 = \$1696.98$

You buy a house for $300000, and you put 10% down. What will your monthly payments be if you get a 30-year mortgage at 8%?

$PMT = \frac{270{,}000 \cdot .08/12}{\left[1 - (1+.08/12)^{-12 \cdot 30}\right]} = \$1981.17$

N= 360
I%= 8
PV= 270000
■PMT= -1981.1643
FV= 0
P/Y= 12
C/Y= 12
PMT:END BEGIN

How much would you have ultimately paid for the house?

$\$30000 + \$1981.17 \cdot 12 \cdot 30 = \$743221.2$

How much interest did you pay?

$\$743221.2 - \$300000 = \$443221.2$

Monthly Balance Table

[For an account/credit card with a starting balance of $500 and an Annual Percentage Rate (APR) of 15%]

Month	Payment	Expenses	Interest	Balance
0	-	-	-	$500
1	$100	$85	$\$500 \cdot .15/12 = \$6.25$ [1]	$491.25 [2]
2	$250	$165	$\$491.25 \cdot .15/12 = \$6.14$	$412.39
3	$200	$40	$\$412.39 \cdot .15/12 = \$5.16$	$257.55
4	$150	-	$\$257.55 \cdot .15/12 = \$3.22$	$110.77

1 - To find the interest charge (finance charge) for each month, take the previous month's balance, and multiply it by the annual interest rate divided by 12 (remember that r/12 is the monthly interest rate).

$$\$500 * \left(\frac{.15}{12}\right) = \$6.25$$

2 – To find the new balance for the month, take the previous month's balance, subtract the payment, add the expenses, and add the interest charge.

$$\$500 - \$100 + \$85 + \$6.25 = \$491.25$$

Credit Card Payoff/Transfer Example

Your current balance on a credit card is $3,750 and the annual percentage rate is 21%. You want to pay the balance off in four and a half years. Assuming that you will make no more purchases with this card and no other costs will be incurred, how much should you pay each month to eliminate the debt in the time you have allotted?

N=	4.5X12 = 54	
I%=	21	
PV=	3750	
*PMT=	-107.91	
FV=	0	
P/Y=	12	
C/Y=	12	
PMT:	END	BEGIN

You will pay $107.71 each month to pay off the balance in 4.5 years

You receive a letter in the mail from a different credit card company offering you better interest rates for 2 years if you agree to transfer the balance and agree to a 4% transfer fee. They will use an APR of 1.99% for the first six months, the next year the APR will be 8.99%, and the final six months of the offer, the APR becomes 15.99%. After the two year period, the APR will be fixed at 49.99%. Is it better to continue with the first plan, or should you transfer the balance?

N=	6	
I%=	1.99	
PV=	3750 +.04(3750)	
PMT=	-107.91	
*FV=	-3288.22	
P/Y=	12	
C/Y=	12	
PMT:	END	BEGIN

The original balance plus the transfer fee

The payment will be the same as before

The FV at the end of this time period is the PV for the next time period

N=	12	
I%=	8.99	
PV=	3288.82	
PMT=	-107.91	
*FV=	-2247.35	
P/Y=	12	
C/Y=	12	
PMT:	END	BEGIN

N=	6	
I%=	15.99	
PV=	2247.35	
PMT=	-107.91	
*FV=	-1763.7	
P/Y=	12	
C/Y=	12	
PMT:	END	BEGIN

N=	30	
I%=	49.99	
PV=	1763.7	
PMT=	-107.91	
*FV=	222.09	
P/Y=	12	
C/Y=	12	
PMT:	END	BEGIN

30 months remain from the original 54 months

Since the FV is positive, you would have overpaid by this amount over the course of the 4.5 years (54 months)

Take the offer!

Exponential Growth/Decay

General Exponential Form: $y = a \cdot b^x$ Doubling Formula: $Q(t) = Q_0 \cdot 2^{t/T_{double}}$ Half-Life Formula: $Q(t) = Q_0 \cdot \left(\frac{1}{2}\right)^{t/T_{half}}$

Rule of 72 (Estimate for doubling or halving time): $\frac{72}{r\%}$

Examples:

Estimate how long it would take for a population to double if it was growing at an annual rate of 6%. *72/6 = 12 years*

In 2000, the population of a city was 35,000. If the population was doubling every 15 years, what would you predict the population would be in 2040?

$$Q(40) = 35,000 \cdot 2^{40/15} = 222,236\, people$$

What fraction will remain after 100 years of a substance that has a half-life of 25 years? $\frac{Q}{Q_0} = \left(\frac{1}{2}\right)^{100/25} = .0625 \text{ or } 6.25\%$