Logarithms (Logs)

Definition of a Logarithm: Logarithms are the inverse functions of exponentials.

$$\log_a M = b \text{ such that } a^b = M \text{ where } M > 0 \text{ and } a > 0 \text{ but } a \neq 1$$

Natural Log: The ***natural log*** is a logarithm whose base is the natural number ***e***. $\log_e M = \ln M$

Zero Identity: For any logarithm of 1 with a positive base, the value is ***always*** zero.

General Form: $\log_a 1 = 0 \text{ where } a > 0$

Multiplication Rule: The log of a ***product*** is equivalent to the ***sum*** of separate logs.

General Form: $\log_a (M \cdot N) \Leftrightarrow \log_a M + \log_a N$

Examples: $\log_{10}(3x) = \log_{10} 3 + \log_{10} x$ $\qquad$ $\log_4 3 + \log_4 2 = \log_4 (3 \cdot 2) = \log_4 6$

Division Rule: The log of a ***quotient*** is equivalent to the ***difference*** of separate logs.

General Form: $\log_a \left(\frac{M}{N}\right) \Leftrightarrow \log_a M - \log_a N$

Examples: $\log_5 \left(\frac{x}{7}\right) = \log_5 x - \log_5 7$ $\qquad$ $\log_5 6 - \log_5 3 = \log_5 \left(\frac{6}{3}\right) = \log_5 2$

Change of Base: The base of a log can be changed into any desired positive value by utilizing the following formula:

General Form: $\log_b M = \frac{\log_a M}{\log_a b} = \frac{\log M}{\log b} = \frac{\ln M}{\ln b}$

Examples: $\log_3 10 = \frac{\log 10}{\log 3} = \frac{1}{\log 3}$ or $\frac{\ln 10}{\ln 3}$ $\qquad$ $\log_2 9 = \frac{\log_{10} 9}{\log_{10} 2}$ or $\frac{\ln 9}{\ln 2}$

Power Rules: Exponents ***inside*** logarithms may be moved outside as a ***multiplier***.

General Form: $\log_a M^k = k \cdot \log_a M$

Examples: $\log_3 x^4 = 4 \log_3 x$ $\qquad$ $\ln \sqrt{x} = \ln x^{1/2} = \frac{1}{2} \ln x$

Cancellation Rule: Taking the log of a number where the base and the number are the same produces a value of 1 or "cancels". Conversely, raising a number to a log with that number as a base produces a value of 1 or "cancels".

General Form: $\log_a a = 1$ $\qquad$ $\ln e^k = k$ $\qquad$ $a^{\log_a k} = k$ $\qquad$ $e^{\ln k} = k$

Examples: $\log_2 2 = 1$ $\qquad$ $\cancel{\log_a a^5} = 5$ $\qquad$ $\cancel{\ln e^{-3}} = -3$ $\qquad$ $\cancel{e^{\ln 2}} = 2$

Combine the rules together: $\ln \frac{x^5 y^3}{e^2} = \ln x^5 + \ln y^3 - \cancel{\ln e^2} = 5 \ln x + 3 \ln y - 2$