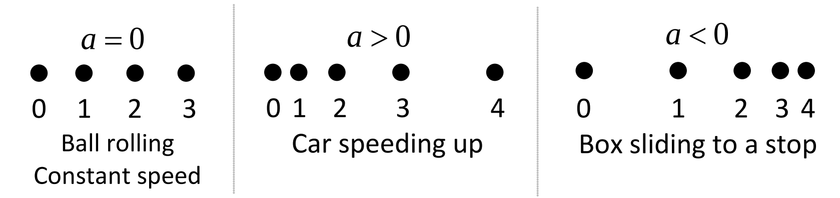
Physics – Mechanics

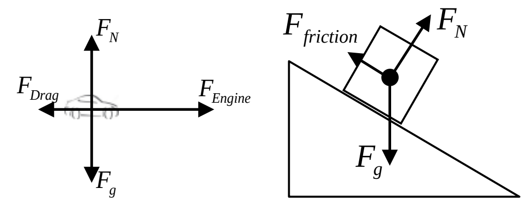
Rule #1: DRAW A PICTURE (Pictorial Representation, Motion Diagrams, Free-Body Diagrams)

Draw a picture of what the question is describing. Use motion diagrams and free-body diagrams to assist you in seeing what is going on in the question.

Motion Diagrams: $\rightarrow +x$



Free-Body Diagrams:



Rule #2: LABEL/LIST YOUR KNOWN VALUES AND THOSE DESIRED (Include units!!)

Label or list all values that were given to you in the problem. Also, include any values that were not explicitly stated, but can be inferred from the problem (gravity occurs in most problems, but goes unsaid, so list $\vec{a}_g = -9.8 \, m/s^2$). Lastly, label those values that you are *seeking as well*.

Motion in One Dimension (Translational Kinematics)

Equations for one-dimensional motion follow, but it is important to note that not all motion is one-dimensional. In order to still use these equations, motion that occurs in multiple dimensions must be broken apart into one-dimensional components. The one-dimensional equations may then be used for each component separately.

Equations of One-Dimensional Motion (x has been used to denote the position in any general direction; s also commonly used)

$$\Delta\vec{x} = \text{area under } \vec{v}\text{-}t \text{ graph } = \int \vec{v} \cdot dt$$

$$\Delta\vec{v} = \text{area under } \vec{a}\text{-}t \text{ graph } = \int \vec{a} \cdot dt$$

$$\vec{v}_{avg} = \text{slope of } \vec{x}\text{-}t \text{ graph } = \frac{\Delta\vec{x}}{\Delta t} = \frac{\vec{x}_f - \vec{x}_0}{t_f - t_0}$$

$$\vec{a}_{avg} = \text{slope of } \vec{v}\text{-}t \text{ graph } = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_0}{t_f - t_0}$$

$$\vec{v}_{inst} = \frac{d\vec{x}}{dt} \qquad \vec{a}_{inst} = \frac{d\vec{v}}{dt}$$

Equation for Constant Velocity (zero acceleration)

$$\vec{x}_f = \vec{v}_0 \cdot \Delta t + \vec{x}_0$$

Equations for Constant Acceleration

$$\vec{x}_f = \frac{1}{2}\vec{a} \cdot (\Delta t)^2 + \vec{v}_0 \cdot \Delta t + \vec{x}_0$$

$$\vec{v}_f = \vec{a} \cdot \Delta t + \vec{v}_0$$

$$\vec{v}_f^{\,2} = \vec{v}_0^{\,2} + 2\vec{a} \cdot \Delta\vec{x}$$

Variables

Δ denotes "change in"
t = Time

$\vec{x}$ = Position in any general direction
$\vec{x}_0$ = Initial Position
$\vec{x}_f$ = Final Position

$\vec{v}$ = Velocity
$\vec{v}_{avg}$ = Average Velocity
$\vec{v}_{inst}$ = Instantaneous Velocity
$\vec{v}_0$ = Initial Velocity
$\vec{v}_f$ = Final Velocity

$\vec{a}$ = Acceleration
$\vec{a}_{avg}$ = Average Acceleration
$\vec{a}_{inst}$ = Instantaneous Acceleration

Projectile Motion

Projectile motion can be simplified into two separate one-dimensional motions: one motion of the object going up & down and a separate motion of the object going left or right. These motions can be considered independently. Once the motions are separated we are free to use the equations for one-dimensional motion on each component.

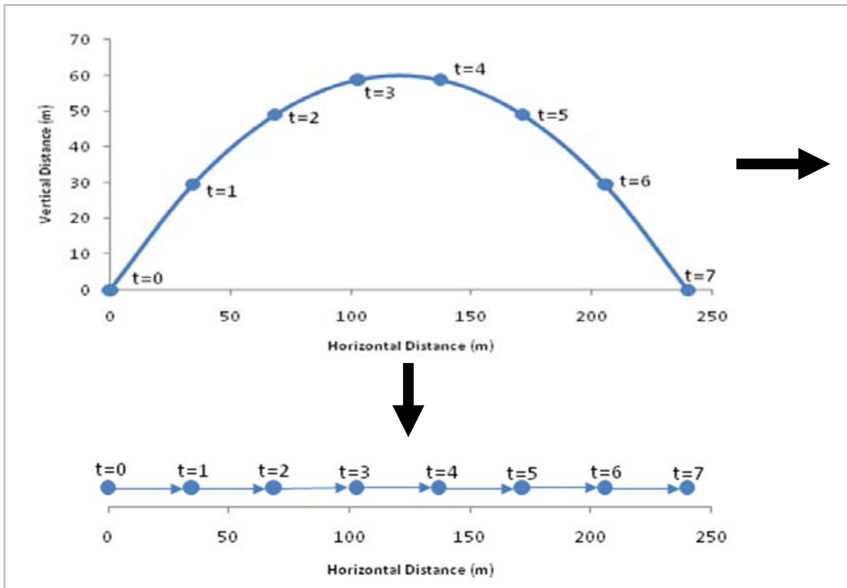


Figure 1

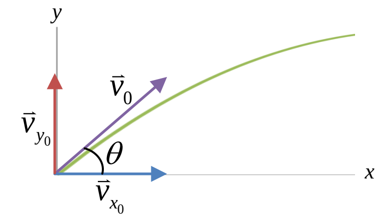


Figure 2

Equations of Projectile Motion

$\vec{v}_{x_0} = v_0 \cdot \cos\theta$

$\vec{v}_{y_0} = v_0 \cdot \sin\theta$

$\vec{a}_x = 0$ (ignoring air resistance)

$\vec{a}_y = \vec{a}_g$

Horizontal Range Equation
(distance traveled when an object launches and lands at same height)

This equation can be derived using equations for one-dimensional motion, equations for projectile motion, and the fact that $y_f = y_0$.

$$x_f - x_0 = \Delta x = \frac{\vec{v}_0^2 \cdot \sin(2\theta)}{a_g}$$

Vertical Range Equation (maximum height of object)

This equation can be derived using equations for one-dimensional motion, equations for projectile motion, and the fact that at the maximum height $\vec{v}_y = 0$.

$$y_f - y_0 = \Delta y = \frac{\vec{v}_0^2 \cdot (\sin\theta)^2}{2a_g}$$

Variables

$\theta =$ Launch Angle off $+x$ axis

$\vec{v}_0 =$ Initial Velocity

$\vec{v}_{x_0} =$ Initial Velocity in x direction

$\vec{v}_{y_0} =$ Initial Velocity in y direction

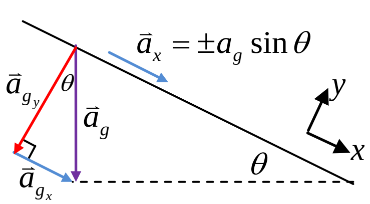
$\vec{a}_x =$ Acceleration in x direction

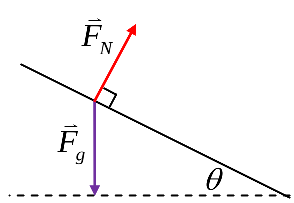
$\vec{a}_y =$ Acceleration in y direction

$\vec{a}_g =$ Acceleration due to gravity

NOTE! $\vec{a}_g$ has magnitude and direction since it is the vector for gravity (direction would be a positive or negative sign). a_g has only the magnitude since it is the *scalar* for gravity (meaning a_g is always positive).

Motion down a Frictionless Inclined Plane





Variables

$\theta =$ Angle of Incline

$\vec{a}_x =$ Acceleration down plane

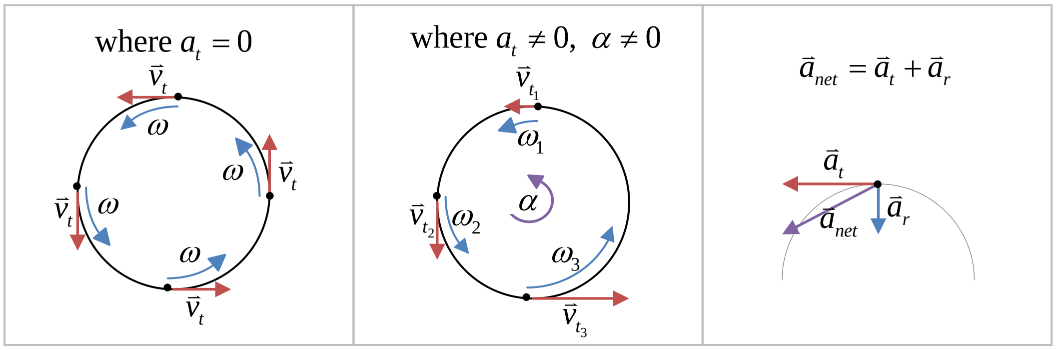
$\vec{a}_g =$ Acceleration due to Gravity

$\vec{a}_{g_x} =$ Gravity in the x direction

$\vec{a}_{g_y} =$ Gravity in the y direction

Circular Motion (Rotational Kinematics)

Circular motion seems like a fairly complicated two-dimensional motion, but when broken down it can be seen that many of the equations and ways we approach circular motion are nearly identical to the one-dimensional equations. When looking at the equations below you may notice that they seem to be one-dimensional equations that just have different variables, and that is exactly what they are. θ replaces s or x or y, ω replaces v, and α replaces a. The difference with circular motion is we have additional equations to solve the additional pieces that exist when motion goes from linear to circular.



General Equations for Circular Motion

$$\Delta s = r \cdot \Delta\theta \qquad a_r = \frac{v_t^2}{r} = \omega^2 r$$

$$v_t = \omega \cdot r = \frac{2\pi r}{T} \qquad a_t = \frac{\Delta v_t}{\Delta t} = \alpha \cdot r$$

$$a_{net} = \sqrt{a_r^2 + a_t^2}$$

$$\Delta\theta = \text{area under } \omega\text{-}t \text{ graph} = \int \omega\, dt$$

$$\Delta\omega = \text{area under } \alpha\text{-}t \text{ graph} = \int \alpha\, dt$$

$$\omega_{avg} = \text{slope of } \theta\text{-}t \text{ graph} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_0}{t_f - t_0}$$

$$\alpha_{avg} = \text{slope of } \omega\text{-}t \text{ graph} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_0}{t_f - t_0}$$

$$\omega_{inst} = \frac{d\theta}{dt} \qquad \alpha_{inst} = \frac{d\omega}{dt}$$

Equation for Constant Angular Velocity/ Uniform Circular Motion (zero angular acceleration)

$$\theta_f = \omega \cdot \Delta t + \theta_0$$

Equations for Constant Angular Acceleration/ Non-Uniform Circular Motion (constant angular acceleration)

$$\theta_f = \frac{1}{2}\alpha\left(\Delta t\right)^2 + \omega_0 \cdot \Delta t + \theta_0$$

$$\omega_f = \alpha \cdot \Delta t + \omega_0$$

$$\omega_f{}^2 = \omega_0{}^2 + 2\alpha \cdot \Delta\theta$$

Variables

r = Radius
T = Period
v_t = Tangential Velocity
a_t = Tangential Acceleration (Δ in speed)
a_r = Radial Acceleration (Δ in direction)

θ = Angular Position
θ_0 = Initial Angular Position
θ_f = Final Angular Position

ω = Angular Velocity
ω_{avg} = Average Angular Velocity
ω_{inst} = Instantaneous Angular Velocity
ω_0 = Initial Angular Velocity
ω_f = Final Angular Velocity

α = Angular Acceleration
α_{avg} = Average Angular Acceleration
α_{inst} = Instantaneous Angular Acceleration
α_0 = Initial Angular Acceleration
α_f = Final Angular Acceleration

Forces

The basic idea behind forces is that a force is a push or pull exerted on an object. We have used equations to show an object's motion, and now we use forces to show ***why*** an object *may be* changing its motion. When looking at forces acting on an object we will tend to separate forces into one-dimensional components just as we did with motion but we can also sum all the forces in those dimensions to see what we call the ***resultant force*** or ***net force***. The idea is that when numerous forces act on an object you can add them all together to see what the ***net force*** is, and this net force determines what kind of change in motion the object experiences. We use the acceleration from this ***net force*** to link forces to our motion equations.

General Force Equations

$$\vec{F} = m \cdot \vec{a}$$

$$\vec{F}_{net_x} = \sum \vec{F}_x = m \cdot \vec{a}_x$$

$$\sum \vec{F}_x = \vec{F}_{1_x} + \vec{F}_{2_x} + \vec{F}_{3_x} + \ldots + \vec{F}_{n_x} = m \cdot \vec{a}_x$$

Specific Force Equations

Friction:

$$f_s \le \mu_s \cdot n$$

$$f_k = \mu_k \cdot n$$

$$f_r = \mu_r \cdot n$$

$$D = C_D \cdot A \cdot v^2$$

$$\vec{F}_{sp} = -k \cdot \Delta\vec{s}$$

$$\vec{F}_g = m \cdot \vec{g}$$

$$\left|F_g\right| = \frac{Gm_1m_2}{r^2}$$

Note on Frictional Forces

When picking which frictional force to use it is important to note when each one should be used. ***Static friction***, $\vec{f}_s$, should be used when the object we are looking at is *not in motion* or is being powered in its roll or braking. ***Kinetic friction***, $\vec{f}_k$, should be used when an object is *moving/sliding* across a surface. ***Rolling friction***, $\vec{f}_r$, should be used when an unpowered object is *rolling* across a surface. Frictional forces always point in the direction ***opposite*** to the motion, or in the case of static friction, in the direction to prevent motion.

Variables

$\vec{F}$ = Force
m = Mass
$\vec{a}$ = Acceleration

$\vec{F}_{net}$ = Net Force
$\vec{F}_x$ = Forces in x-direction

$\vec{F}_{sp}$ = Spring Force
k = Spring Constant
Δs = Distance Stretched/Compressed

$\vec{n}$ = Normal Force
$\vec{f}_s$ = Static Friction
μ_s = Coefficient of Static Friction

$\vec{f}_k$ = Kinetic Friction
μ_k = Coefficient of Kinetic Friction

$\vec{f}_r$ = Rolling Friction
μ_r = Coefficient of Rolling Friction

$\vec{D}$ = Drag Force (opposite to motion)
A = Cross-Sectional Area ($\perp$ to motion)
C_D = Coefficient of Drag

Momentum & Impulse

Momentum can be thought of as a quantity that represents how difficult it is to stop an object in motion or change an object's direction of motion. Our main use for momentum comes from the fact that *in a closed system* the *total momentum is constant* (***Conservation of Momentum***). This fact allows us to have a better understanding of the interaction of objects, particularly in collisions and explosions. What seem like chaotic interactions in collisions and explosions can be broken into parts, and so long as our system is closed, the sum of the momenta before the event is equal to the sum of the momenta after the event. If the system is not closed, then we have to take into consideration any outside forces taking or giving momentum to our system. ***Impulse*** is the change in momentum for an object and is equal to the product of a force and the duration of time that it is applied.

Momentum & Impulse Equations

$$\vec{p} = m \cdot \vec{v}$$

$$\vec{J}_x = \Delta\vec{p}_x \approx \vec{F}_{x_{avg}} \cdot \Delta t$$

$$\vec{J}_x = \int_{t_0}^{t_f} \vec{F}_x(t) \cdot dt = \text{area under } \vec{F}\text{-}t \text{ graph}$$

Variables

$\vec{p}$ = Momentum
$\vec{J}$ = Impulse
$\vec{F}_{x_{avg}}$ = Average Force in x-direction
Δt = Change in Time

Conservation of Momentum

$$\sum \vec{p}_{x_0} = \sum \vec{p}_{x_f} \qquad \vec{p}_{x1_0} + \vec{p}_{x2_0} + \vec{p}_{x3_0} + \ldots + \vec{p}_{xn_0} = \vec{p}_{x1_f} + \vec{p}_{x2_f} + \vec{p}_{x3_f} + \ldots + \vec{p}_{xn_f}$$

Remember: You can only use the ***conservation of momentum*** if the system is "isolated" or "closed". This means that you can only use the ***conservation of momentum*** if there are no outside forces that are adding or taking momentum from the system.

Energy, Work & Power

We can think of systems as having ***energy***, and if there are no outside forces on these systems, then the energy is conserved much in the way momentum is conserved. Just as an outside force can change the momentum of the system, an outside force can *change* the energy of a system through what we call ***work***. ***Power*** is the rate at which energy is transferred of transformed.

Energy & Work Equations

$$K = \frac{1}{2} m \cdot v^2$$

$$U_{sp} = \frac{1}{2} k \cdot (\Delta s)^2$$

$$U_g = m \cdot g \cdot h \text{ or } m \cdot g \cdot "y" \qquad \Delta U_g = m \cdot g \cdot \Delta y$$

$$\Delta E_{th} = \left| f \cdot \Delta r \right|$$

$$W = \int_{x_0}^{x_f} F_x \cdot dx = \text{Area under } F\text{-}x \text{ curve}$$

$$W = \vec{F} \cdot \Delta \vec{r}, \text{ if } F \text{ is constant and straight-line motion}$$

$$W = \vec{F} \cdot \Delta r \cos\theta$$

Variables

K = Kinetic Energy
U_{sp} = Elastic Potential Energy (Spring)
k = Spring Constant
Δr = Change in position in any general direction
U_g = Gravitational Potential Energy
h = Height ~ Δy = Change in Vertical Position
ΔE_{th} = Change in Thermal Energy
W = Work
$\Delta \vec{r}$ = Distance Traveled in Same Direction as Force
W_{ext} = Work External

Conservation of Energy

$$K_0 + U_0 + W_{ext} = K_f + U_f + \Delta E_{th}$$

The conservation of energy is useful in many situations because, unlike the conservation of momentum, we can still use the conservation of energy if there are outside forces. Outside forces are taken into account by work done on the system.

Power

$$P = \frac{\Delta E_{sys}}{\Delta t} = \frac{dE_{sys}}{dt}$$

$$P = \vec{F} \cdot \vec{v}_{inst} = F \cdot v \cos\theta$$

Variables

P = Power
E_{sys} = Energy of the System
t = Time
F = Force
v = Velocity

Rotation of a Rigid Body

The following rotational motion equations can be used when you have a rigid body that is being revolved around a fixed point.

Rotational Motion Equations

$$M = \sum_i m_i = m_1 + m_2 + m_3 + \ldots$$

$$\vec{X}_{cm} = \frac{1}{M} \sum_i \left(m_i \cdot \vec{x}_i \right) = \frac{m_1\vec{x}_1 + m_2\vec{x}_2 + m_3\vec{x}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$

$$\vec{X}_{cm} = \frac{1}{M} \int \vec{x} \cdot dm$$

$$K_{rot} = \frac{1}{2} I \cdot \omega^2$$

$$I = \int r^2 \cdot dm = \sum m_i r_i^2 \text{ (for point masses)}$$

$$I = I_{cm} + M \cdot d^2$$

Variables

M = Total Mass
X_{cm} = Center of Mass in any general direction
K_{rot} = Rotational Kinetic Energy
ω = Angular Velocity
r = Radius
I_{cm} = Inertia at Center of Mass
d = Distance between Axis of Rotation and Center of Mass
I = Inertia at distance d from Center of Mass about parallel axis

Because the integral to find the inertia about a center of mass can be very difficult to solve, most classes do not require the calculation. General equations for the inertia of different objects will be provided to you or can be found in your text.

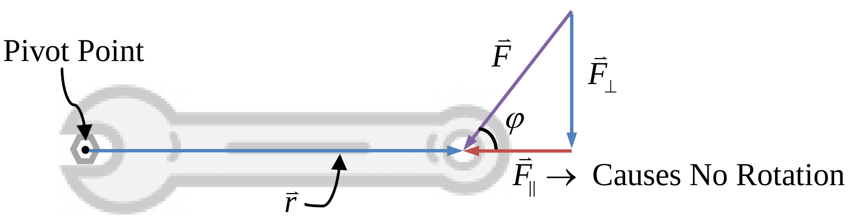
Torque

Torque can be thought of as the ***rotational*** equivalent of force. So for an example when you push a door open you are applying a force to the door, this force exerts a torque around the pivot point (in this case is the hinges) which will cause the door to open. Positive torque provides counterclockwise rotation. Negative torque provides clockwise rotation.

Torque Equations

$\tau = \vec{r} \times \vec{F}$ $\qquad \tau = r \cdot F_\perp$

$\tau_{net} = I \cdot \alpha$ $\qquad \tau = r \cdot F \cdot \sin\varphi$



Variables

$\tau =$ Torque

$r =$ Distance from Pivot Point

$F_\perp =$ Force perpendicular to $\vec{r}$

$\varphi =$ Angle between vectors $\vec{r}$ and $\vec{F}$ (Extended to meet one another)

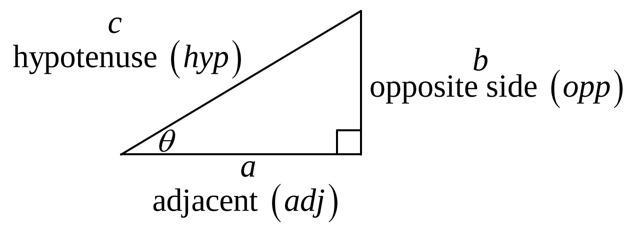
$I =$ Moment of Inertia

$\alpha =$ Angular Acceleration

Useful Trigonometric Equations

$\sin\theta = \frac{opp}{hyp}$ $\qquad \cos\theta = \frac{adj}{hyp}$

$\tan\theta = \frac{opp}{adj}$ $\qquad c^2 = a^2 + b^2$



Constants

$M_e =$ Mass of Earth $= 5.98 \times 10^{24}\, kg$

$R_e =$ Radius of Earth $= 6.37 \times 10^{6}\, m$

$M_{moon} =$ Mass of Moon $= 7.36 \times 10^{22}\, kg$

$R_{moon} =$ Radius of Moon $= 1.74 \times 10^{6}\, m$

$R_{EO} =$ Radius of Earth Orbit $= 1.50 \times 10^{11}\, m$

$g = a_g = -9.81\, m/s^2 = -32\, ft/s^2$

$G = 6.67 \times 10^{-11}\, Nm^2/kg^2$

$v_{sound} =$ Speed of sound in air $= 343\, m/s$

$m_p =$ Mass of a proton or neutron $= 1.67 \times 10^{-27}\, kg$

$m_e =$ Mass of an electron $= 9.11 \times 10^{-31}\, kg$

$\varepsilon_0 =$ Permittivity constant $= 8.85 \times 10^{-12}\, C^2/Nm^2$

$K =$ Coulomb's law constant $\frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^{9}\, Nm^2/C^2$

$\mu_0 =$ Permeability constant $= 1.26 \times 10^{-6}\, Tm/A$

$e =$ Fundamental unit of charge $= 1.60 \times 10^{-19}\, C$

$c =$ Speed of light in a vacuum $= 3.00 \times 10^{8}\, m/s$

Coefficients

Material	***Static*** μ_s	***Kinetic*** μ_k	***Rolling*** μ_r
Rubber on concrete	1.00	0.80	0.02
Steel on steel (dry)	0.80	0.60	0.002
Steel on steel (lubricated)	0.10	0.05	
Wood on wood	0.50	0.20	
Wood on snow	0.12	0.06	
Ice on ice	0.10	0.03	

Metal	***Resistivity(Ωm)***	***Conductivity(1/Ωm)***
aluminum	2.8×10^{-8}	3.5×10^{7}
copper	1.7×10^{-8}	6.0×10^{7}
gold	2.4×10^{-8}	4.1×10^{7}
iron	9.7×10^{-8}	1.0×10^{7}
silver	1.6×10^{-8}	6.2×10^{7}
tungsten	5.6×10^{-8}	1.8×10^{7}

Conversions

1 mile = 5280 feet = 1609 meters = 1.609 kilometers

1 inch = 2.54 centimeters

1 hour = 60 minutes = 3600 seconds

1 revolution = 360° = 2π radians

$1\, m/s = 2.24\, mi/hr = 3.28\, ft/s$

$1\, eV = 1.60 \times 10^{-19}\, J$

$1\, u = 1.66 \times 10^{-27}\, kg$

$1\, kg \approx 2.2 lbs$ on Earth