

Pivoting By Hand

1. Set up augmented matrix with slack variables.

Minimizing

$$Z = 4x + 5y$$

Must set Objective Function = 0 in order to put into augmented matrix

$$\begin{array}{r} Z = 4x + 5y \\ -4x - 5y -4x -5y \\ \hline -4x - 5y + Z = 0 \end{array} \quad \begin{array}{|l} Z \text{ must be} \\ \text{Positive} \end{array}$$

Constraints

$$x + 2y \geq 80$$

$$3x + y \geq 75$$

Constraints must be \geq in order to put into augmented matrix

EX.
 $\frac{10x - 5y \leq 3}{-1 \quad -1 \quad -1}$ (Divide everything by -1 to flip sign)
 $-10x + 5y \geq -3$

x	y	s ₁	s ₂	z	#
1	2	1	0	0	80
3	1	0	1	0	75
-4	-5	0	0	1	0

Maximizing

$$Z = 10x + 8y$$

Must set Objective Function = 0 in order to put into augmented matrix

$$\begin{array}{r} Z = 10x + 8y \\ -Z \\ \hline 0 = 10x + 8y - Z \end{array} \quad \begin{array}{|l} Z \text{ must be} \\ \text{Negative} \end{array}$$

Constraints

$$4x + 2y \leq 80$$

$$x + 2y \leq 50$$

Constraints must be \leq in order to put into augmented matrix

EX.
 $\frac{10x - 5y \geq 3}{-1 \quad -1 \quad -1}$ (Divide everything by -1 to flip sign)
 $-10x + 5y \leq -3$

x	y	s ₁	s ₂	z	#
4	2	1	0	0	80
1	2	0	1	0	50
10	8	0	0	-1	0

2. Find the Pivot Column.

Minimizing

The largest negative number in the bottom row tells your pivot column.

x	y	s ₁	s ₂	z	#
1	2	1	0	0	80
3	1	0	1	0	75
-4	-5	0	0	1	0

Maximizing

The largest positive number in the bottom row tells your pivot column.

x	y	s ₁	s ₂	z	#
4	2	1	0	0	80
1	2	0	1	0	50
10	8	0	0	-1	0

3. Find the Pivot Row.

The smallest positive ratio of solution # to the number in the pivot column.

Minimizing

x	y	s ₁	s ₂	z	#
1	2	1	0	0	80
3	1	0	1	0	75
-4	-5	0	0	1	0

80/2 = 40
75/1 = 75

Maximizing

x	y	s ₁	s ₂	z	#
4	2	1	0	0	80
1	2	0	1	0	50
10	8	0	0	-1	0

80/4 = 20
50/1 = 50

4. Get pivot # to be 1 by dividing the pivot # by itself along with every number in the pivot row. Bring down the rest of the numbers.

<p style="text-align: center; margin: 0;"><u>Minimizing</u></p> <table border="1" style="margin: 0 auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 5px;">X</th> <th style="padding: 5px;">y</th> <th style="padding: 5px;">s₁</th> <th style="padding: 5px;">s₂</th> <th style="padding: 5px;">z</th> <th style="padding: 5px;">#</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;">1</td> <td style="padding: 5px; background-color: #90EE90;">2</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">80</td> </tr> <tr> <td style="padding: 5px;">3</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">75</td> </tr> <tr style="border-top: 1px solid black;"> <td style="padding: 5px;">-4</td> <td style="padding: 5px;">-5</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">0</td> </tr> </tbody> </table> <table border="1" style="margin: 0 auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 5px;">X</th> <th style="padding: 5px;">y</th> <th style="padding: 5px;">s₁</th> <th style="padding: 5px;">s₂</th> <th style="padding: 5px;">z</th> <th style="padding: 5px;">#</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;">R1÷2</td> <td style="padding: 5px; background-color: #90EE90;">1/2</td> <td style="padding: 5px; background-color: #90EE90;">1</td> <td style="padding: 5px;">1/2</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">40</td> </tr> <tr> <td style="padding: 5px;">3</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">75</td> </tr> <tr style="border-top: 1px solid black;"> <td style="padding: 5px;">-4</td> <td style="padding: 5px;">-5</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">0</td> </tr> </tbody> </table>	X	y	s ₁	s ₂	z	#	1	2	1	0	0	80	3	1	0	1	0	75	-4	-5	0	0	1	0	X	y	s ₁	s ₂	z	#	R1÷2	1/2	1	1/2	0	40	3	1	0	1	0	75	-4	-5	0	0	1	0	<p style="text-align: center; margin: 0;"><u>Maximizing</u></p> <table border="1" style="margin: 0 auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 5px;">X</th> <th style="padding: 5px;">y</th> <th style="padding: 5px;">s₁</th> <th style="padding: 5px;">s₂</th> <th style="padding: 5px;">z</th> <th style="padding: 5px;">#</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px; background-color: #90EE90;">4</td> <td style="padding: 5px;">2</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">80</td> </tr> <tr> <td style="padding: 5px;">1</td> <td style="padding: 5px;">2</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">50</td> </tr> <tr style="border-top: 1px solid black;"> <td style="padding: 5px;">10</td> <td style="padding: 5px;">8</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">-1</td> <td style="padding: 5px;">0</td> </tr> </tbody> </table> <table border="1" style="margin: 0 auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 5px;">X</th> <th style="padding: 5px;">y</th> <th style="padding: 5px;">s₁</th> <th style="padding: 5px;">s₂</th> <th style="padding: 5px;">z</th> <th style="padding: 5px;">#</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;">R1÷4</td> <td style="padding: 5px; background-color: #90EE90;">1/4</td> <td style="padding: 5px; background-color: #90EE90;">1</td> <td style="padding: 5px;">1/2</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">20</td> </tr> <tr> <td style="padding: 5px;">1</td> <td style="padding: 5px;">2</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">50</td> </tr> <tr style="border-top: 1px solid black;"> <td style="padding: 5px;">10</td> <td style="padding: 5px;">8</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">-1</td> <td style="padding: 5px;">0</td> </tr> </tbody> </table>	X	y	s ₁	s ₂	z	#	4	2	1	0	0	80	1	2	0	1	0	50	10	8	0	0	-1	0	X	y	s ₁	s ₂	z	#	R1÷4	1/4	1	1/2	0	20	1	2	0	1	0	50	10	8	0	0	-1	0
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5. Using the pivot # get the other number in the pivot column to be 0.

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6. Am I done?

<p style="text-align: center; margin: 0;"><u>Minimizing</u></p> <p style="margin: 0;">If there are still <u>negative</u> numbers in the bottom row repeat steps 2 - 5 until all numbers are <u>positive</u>.</p>	<p style="text-align: center; margin: 0;"><u>Maximizing</u></p> <p style="margin: 0;">If there are still <u>positive</u> numbers in the bottom row repeat steps 2 - 5 until all numbers are <u>negative</u>.</p>
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7. Solutions are in the Matrix.

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<p style="margin: 0;">Basic Variable: The variable that is in the column with a 1 and the rest of the numbers are 0. In both of these solutions the x and y variables are considered basic.</p>
<p style="margin: 0;">Nonbasic Variable: The variable that is in the column with more than one number other than 0. In this case S₁ and S₂ are nonbasic.</p>

<p style="margin: 0;">S₁ and S₂ are = 0 because there is more than 1 number in that column.</p>
