

# Probability and “Number of Ways”

**Event** – any unique occurrence or situation. (ex: getting a heads on a coin flip, the weather being rainy, your favorite sports team winning the championship, etc.)

**Probability** – the likelihood that a particular event will occur. Probabilities can be written as fractions (1/4), decimals (.25), percentages (25%), or ratios/odds (1:3). Decimal values for probabilities will exist between 0 and 1 (ex: .12, .97, etc.) The sum (total) of the probabilities of all possible events is 1 or 100%. The probability of a single event is sometimes referred to as a “**marginal**” probability.

$$P(A) = \frac{\text{Probability of event A}}{\text{total \# of times any event can occur}} = \frac{\text{\# of times the event A occurs}}{\text{total \# of times any event can occur}}$$

The **complement** of an event is the sum of all of the events that can occur, but not including the event that you are finding the probability of. In any instance where it would be easier to calculate the probability of the complement than finding the probability of the event itself, you should calculate the probability of the complement and then subtract that number from 1 (because the probability of all events must add up to 1).

$$P(A) = 1 - P(\text{not } A)$$

Question: What is the probability of rolling a 2 or greater on a fair-sided die?

Answer: The opposite of rolling a two or greater is rolling a 1.

$$P(1) = 1/6, \text{ so } P(2 \text{ or } >) = 1 - 1/6 = 5/6$$

**Independent Events** – events that do not depend on one another. (ex: getting a heads on a coin flip, and rolling a 6 on a die.)

The **intersection** of two events is the probability that two events occur at the same time. This is also referred to as a “**joint**” probability. Typically, when the word and is used, it implies that you should **multiply** the two or more probabilities together for **independent** events.

$$P(A \text{ and } C) = P(A \cap C) = P(A) * P(C)$$

The **union** of two events is the probability that one event or another event occur. Typically, when the word or is used, it implies that you should add the two or more probabilities together for independent events.  $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$

For dependent events:  $P(A \text{ or } C) = P(A \cup C) = P(A) + P(C) - P(A \cap C)$

**“Conditional”** probabilities are those that are calculated using given information.

$P(A|C) = \frac{P(A \cap C)}{P(C)}$  This is the probability of event A occurring *given* that event C has already occurred.

**Classical Probability** – the measure of likelihood that an event will occur based on the ratio of the number of favorable outcomes to the total number of possible outcomes.

(This is the most widely used probability. There is a 1 in 6 chance of rolling a 3 with a fair die.)

**Experimental Probability** – the likelihood that an event will occur based off of experimental data.

(It rained on New Year's Day 4 of the last 5 years, so there is an 80% chance that it will rain this year.)

**Subjective Probability** – the measure of a person's belief that an outcome will occur.

(I think there's a 60% chance that the Cubs will win the World Series.)

**Permutation** – A permutation is the total number of ways that you can arrange  $r$  things from  $n$  total things. In a permutation, order **does** matter. (ex: ABC, ACB, BAC, and CBA are all different)

$${}_n P_r = \frac{n!}{(n-r)!}$$

**Combination** – A combination is the total number of ways that you can choose  $r$  things from  $n$  total things. In a combination, order **does not** matter. (ex: ABC is no different from CAB or BCA)

$${}_n C_r = \frac{n!}{r!(n-r)!}$$

**License Plates:** How many license plates can be made with 3 letters followed by 3 numbers if repetition is allowed?

$$\underline{26} \quad \underline{26} \quad \underline{26} - \underline{10} \quad \underline{10} \quad \underline{10} = 26*26*26*10*10*10 = 17,576,000 \text{ license plates}$$

If repetition is **NOT** allowed:  $\underline{26} \quad \underline{25} \quad \underline{24} - \underline{10} \quad \underline{9} \quad \underline{8} = 26*25*24*10*9*8 = 11,232,000$  plates

Zip Codes: How many **even** 5-digit zip codes are available if they *cannot* start with a zero?

$$\underline{9} \quad \underline{10} \quad \underline{10} \quad \underline{10} \quad \underline{5} = 9*10*10*10*5 = 45,000 \text{ zip codes}$$

How many different ways can the 11 players on a basketball team be on the court in a group of 5?

You should use a "Combination" since *order doesn't matter*:

$${}_n C_r = {}_{11} C_5 = \frac{11!}{5!(11-5)!} = \frac{11!}{5!6!} = 462$$

How many different ways can the starting 5 players be called off the bench out of all 11 players?

You could use the license plate method. There are 11 players available for the first spot, but only 10 left for the second spot, and so on:  $11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 = 55440$

or you could use a "Permutation" since *order does matter*:

$${}_n P_r = {}_{11} P_5 = \frac{11!}{(11-5)!} = \frac{11!}{6!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 = 55440$$