Quadratic Functions

Standard Form MAT12X/15X Helpful for identifying the vertical intercept	**Factored Form** MAT12X/15X Helpful for identifying horizontal intercepts	**Vertex Form** MAT15X Helpful for identifying the vertex
$\boldsymbol{y = ax^2 + bx + c}$ Example: $y = 2x^2 - 4x - 6$	$\boldsymbol{y = a(x - x_1)(x - x_2)}$ Example: $y = 2(x + 1)(x - 3)$	$\boldsymbol{y = a(x - h)^2 + k}$ Example: $y = 2(x - 1)^2 - 8$

Axis of symmetry – *the vertical line that splits the parabola in half, written as an equation x = #. This value is the x-coordinate of the vertex.*

$\boldsymbol{x = \frac{-b}{2a}}$ $x = \frac{-(-4)}{2(2)}$ $= \frac{4}{4}$ Axis of symmetry: $x = 1$	$\boldsymbol{x = \frac{x_1 + x_2}{2}}$ $x = \frac{-1 + 3}{2}$ *midpoint of zeros* $= \frac{2}{2}$ Axis of symmetry: $x = 1$	$\boldsymbol{x = h}$ h is 1 Axis of symmetry $x = 1$

Vertex – *also called the turning point of the graph. This is where the maximum/minimum occur. To find the vertex, substitute the x value from the axis of symmetry into the function to evaluate the y value. Write as an ordered pair (x, y).*

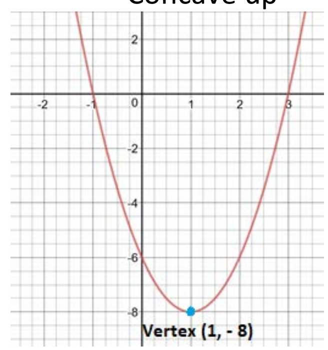
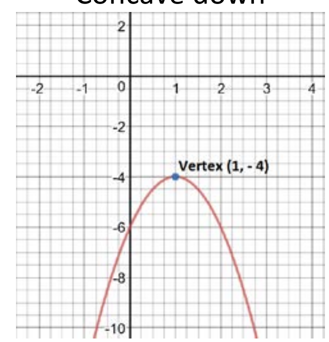
$\boldsymbol{\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)}$ $f(1) = 2(1)^2 - 4(1) - 6$ $= 2 - 4 - 6$ $f\left(\frac{-b}{2a}\right) = -8$ Vertex: $(1, -8)$	$\boldsymbol{\left(\frac{x_1+x_2}{2}, f\left(\frac{x_1+x_2}{2}\right)\right)}$ $f(1) = 2((1) + 1)((1) - 3)$ $= 2(2)(-2)$ $f\left(\frac{x_1+x_2}{2}\right) = -8$ Vertex: $(1, -8)$	$\boldsymbol{(h, k)}$ h is 1, k is -8 Vertex: $(1, -8)$

Horizontal Intercepts – *occur where the graph is on the x-axis (y value is zero). Set the equation equal to zero and solve. If equation is in standard form, you have to use the quadratic formula to solve algebraically.*

$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$ $0 = ax^2 + bx + c$ $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-6)}}{2(2)}$ $= \frac{4 \pm \sqrt{16 + 48}}{4}$ $x = \frac{4+8}{4} = 3$ and $x = \frac{4-8}{4} = -1$ Horizontal intercepts: $(-1,0)$ and $(3,0)$	$\boldsymbol{(x_1, 0), (x_2, 0)}$ $0 = a(x - x_1)(x - x_2)$ $0 = 2(x + 1)(x - 3)$ $0 = x + 1$ and $0 = x - 3$ $x = -1$ $x = 3$ Horizontal intercepts: $(-1,0)$ and $(3,0)$	$\boldsymbol{0 = a(x - h)^2 + k}$ $0 = 2(x - 1)^2 - 8$ $8 = 2(x - 1)^2$ $4 = (x - 1)^2$ $\pm\sqrt{\mathbf{4}} = \sqrt{(x - 1)^2}$ $\mathbf{2} = x - 1$ and $\mathbf{-2} = x - 1$ $x = 3$ $x = -1$ Horizontal intercepts: $(-1,0)$ and $(3,0)$

Vertical Intercept – *occurs where the graph crosses the y-axis (x value is zero). Substitute zero for the input and solve for the y value.*

$(\mathbf{0}, \boldsymbol{c}) * \textbf{substitute 0 for } \boldsymbol{x}$ $y = 2(0)^2 - 4(0) - 6$ $= 0 - 0 - 6$ $= -6$ Vertical intercept: $(0, -6)$	$* \textbf{Substitute 0 for } \boldsymbol{x}$ $y = 2(0+1)(0-3)$ $= 2(1)(-3)$ $= -6$ Vertical intercept: $(0, -6)$	$* \textbf{substitute 0 for } \boldsymbol{x}$ $y = 2(0-1)^2 - 8$ $= 2(1) - 8$ $= -6$ Vertical intercept: $(0, -6)$

In $y = ax^2 + bx + c$, what do a, b, and c mean? **a:** one half of the rate of change in the rate of change (1/2 of the second difference) **b:** the instantaneous rate of change at x=0 (initial rate of change) **c:** the value of y at x=0 (initial value/vertical intercept)	Concave up 	Concave down 
Domain: all possible inputs	all real numbers also written as: $-\infty < x < \infty$	
Range Look at the y coordinate of the vertex. Does the graph go up from there or down?	$\boldsymbol{y \geq k}$ Range: $y \geq -8$ also written as: $-8 \leq y < \infty$	$\boldsymbol{y \leq k}$ Range: $y \leq -4$ also written as: $-\infty < y \leq -4$
Intervals of Increasing Look at the x coordinate of the vertex. Does the graph increase on the left or right of that?	$(\boldsymbol{h}, \infty)$ increasing: $(1, \infty)$ to the *right* of the vertex also written as: $x > 1$	$(-\infty, \boldsymbol{h})$ increasing: $(-\infty, 1)$ to the *left* of the vertex also written as: $x < 1$
Intervals of decreasing Look at the x coordinate of the vertex. Does the graph decrease on the left or right of that?	$(-\infty, \boldsymbol{h})$ decreasing: $(-\infty, 1)$ to the *left* of the vertex also written as: $x < 1$	$(\boldsymbol{h}, \infty)$ decreasing: $(1, \infty)$ to the *right* of the vertex also written as: $x > 1$