MAT12X – Intermediate Algebra

Workshop I – Quadratic Functions

CHANDLER-GILBERT
COMMUNITY COLLEGE
LEARNING CENTER

Overview

Workshop I

- Quadratic Functions
  - General Form
  - Domain and Range
  - Some of the effects of the leading coefficient “a”
  - The vertex
  - Increasing and decreasing intervals
  - Vertex form
  - Solving quadratic inequalities

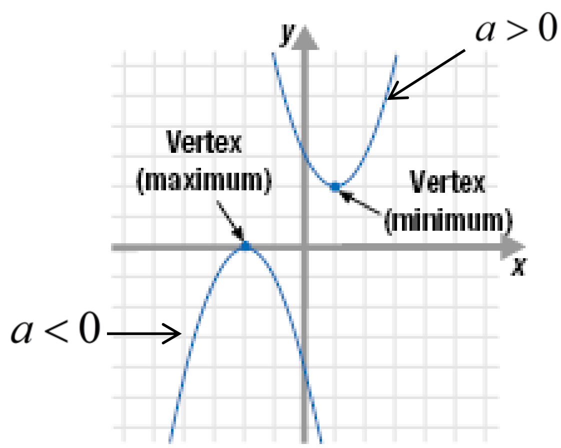
General Form: $f(x) = ax^2 + bx + c$, where $a \neq 0$.

In order to have general form, all terms must be on one side of the equation. Which of the following quadratic equations are in general form?

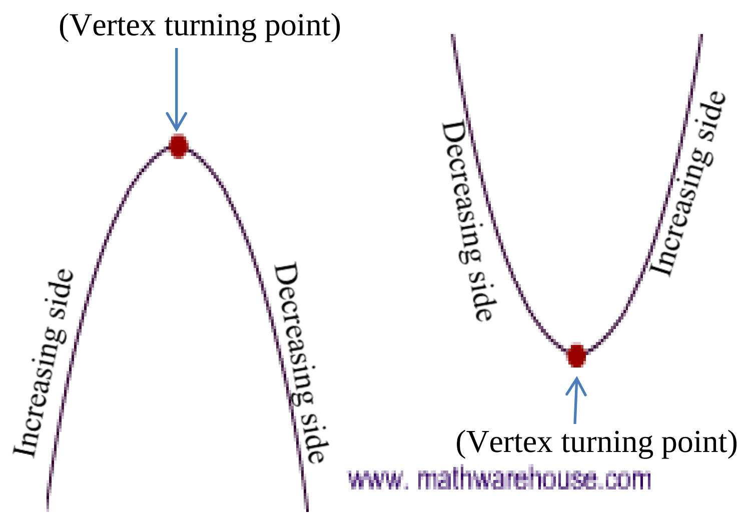
A. $2 = -3x^2 + 2x$
B. $4x^2 - x - 4 = 0$
C. $.5x^2 = 2x - 8$
D. $-.3x^2 = 0$
E. $2x^2 - 9x = 0$

The domain for all quadratic functions is all real numbers; however, the range is dependent upon the vertex (turning point) of the parabola. The range will either be $f(x) \geq$ the output at the vertex (if the vertex is a minimum) or $f(x) \leq$ the output at the vertex (if the vertex is a maximum).

Depending upon the sign of the coefficient "a", the parabola will open upwards ($a > 0$) or downwards ($a < 0$). Which means that the vertex will be a minimum point when the parabola opens upward ($a > 0$) or it will be a maximum when the parabola opens downwards ($a < 0$).

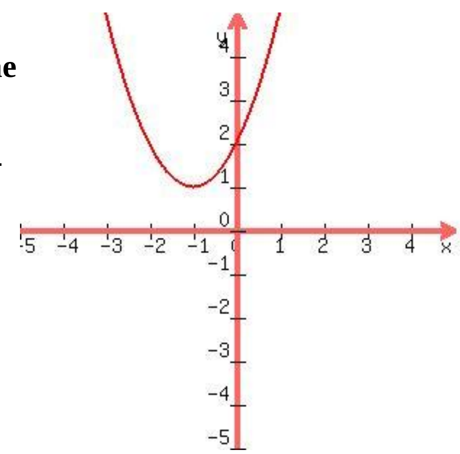


All quadratic functions will be increasing or decreasing on either side of the vertex. Whether it is increasing or decreasing is determined by which way the parabola opens (either opens upward or downward).



When stating where the quadratic function is increasing or decreasing we use the interval of the input to do this. For example, the graph below is a parabola opening upward with a vertex at (-1, 1). Therefore, it is decreasing on the interval of the input where the input (x) is less than -1. Stated symbolically, we write: $x < -1$ or $-\infty < x < -1$ for inequality notation, or $(-\infty, -1)$ in interval notation. Both are different ways of saying the same thing! **Where is the parabola increasing?**

The function is increasing on the interval of the input where the input is ____________________



Inequality notation:

Interval notation:

The domain for this function is all real numbers, but the range is $f(x) \geq 1$ in inequality notation and $[1, \infty)$ in interval notation. Notice that when using the < or > symbols in inequality notation, we use "( )" in interval notation and when we use the $\leq or \geq$ symbols in inequality notation, we use "[ ]" in interval notation. The brackets [ ] indicate that the number next to it is included in the interval just like the less than or equal to symbol ($\leq$) or the greater than or equal to symbol ($\geq$). When the number is not included, we use parentheses ( ) to indicate this just like the less than symbol (<) and the greater than symbol (>).

To find the range of a quadratic equation (remember the domain is always all real numbers!) algebraically, we must find the output of the vertex point. We can do this by using the fact that the input point of the vertex will be: $\frac{-b}{2a}$. Once we find the input value of the vertex, we can determine the output using the function: $f\left(\frac{-b}{2a}\right)$. [The symbol $f\left(\frac{-b}{2a}\right)$ means to "plug in" the value of $\frac{-b}{2a}$ into the function to determine its output].

Find the domain and range of the function and determine the intervals on which the function is increasing and decreasing.

$f(x) = x^2 - 3x - 4$

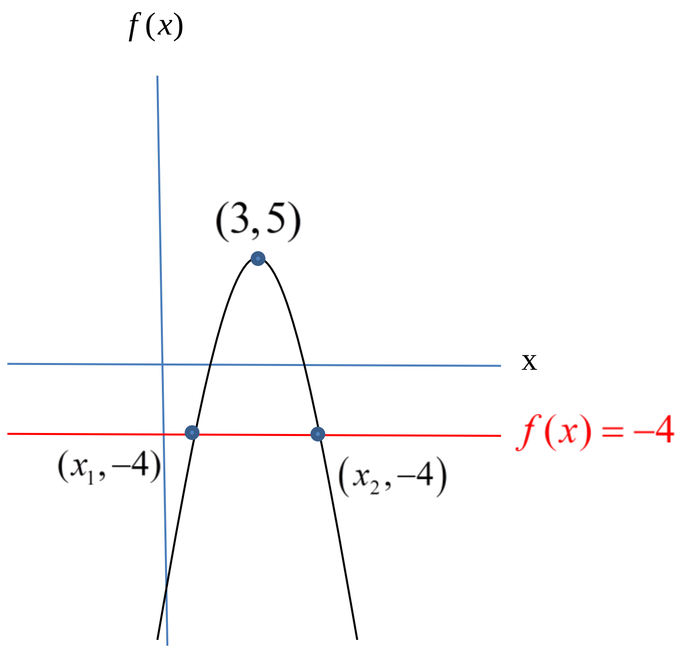
The domain is all real numbers [in interval notation we write it as: $(-\infty, \infty)$]. The input value of the vertex is : $x = \frac{-b}{2a} = \frac{-(-3)}{2(1)} = \frac{3}{2}$.

So the output value of the vertex is:
$f\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) - 4 = \left(\frac{3^2}{2^2}\right) - \frac{3\times3}{2} - 4 = \left(\frac{3\times3}{2\times2}\right) - \frac{9}{2} - 4 = \frac{9}{4} - \frac{9}{2} - 4 = \frac{9}{4} - \frac{18}{4} - 4 = -\frac{9}{4} - 4 = -\frac{9}{4} - \frac{16}{4} = -\frac{25}{4}$
. Which means the range is $f(x) \geq -\frac{25}{4}$. The interval of decrease is when the input is less than $\frac{3}{2}$, or in inequality notation: $x < \frac{3}{2}$; and in interval notation: $\left(-\infty, \frac{3}{2}\right)$. The interval of increase is when the input is greater than $\frac{3}{2}$, or in inequality notation: $x > \frac{3}{2}$; and in interval notation: $\left(\frac{3}{2}, \infty\right)$.

You try it! Find the domain and range of the function and determine the intervals on which the function is increasing and decreasing.

$h(x) = -2x^2 + 12x - 13$

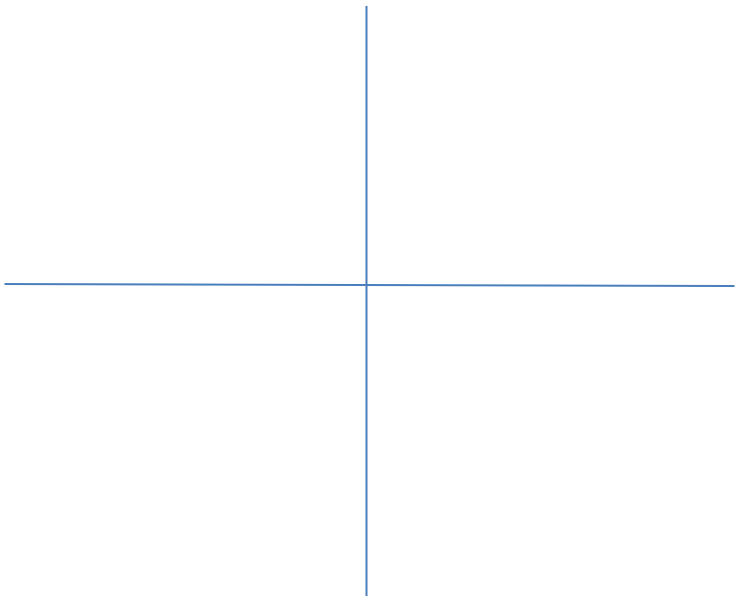
What if the question is: find all the values of the input for which $-2x^2+12x-13>-4$. For these types of questions, graphing them makes it much easier! Using our graphing calculators, we will graph $-2x^2+12x-13$ in Y_1 and -4 in Y_2. You should see something like this (without the coordinates):



We need to find the two input values for which the output is -4. These will tell us what inputs make the output equal to -4 not where the output is greater than -4, but by looking at the graph, we can see any inputs that are greater than x_1 but are also less than x_2 will produce outputs that are greater than -4. This is why we need to find them! Using the intersection function of your calculator, we find $x_1 \approx .879$ and $x_2 \approx 5.121$. So in order for the function's output to be greater than -4 we must select inputs which are greater than .879 AND less than 5.121. To write this statement symbolically, we write: $.879 < x < 5.121$ using inequality notation or $(.879, 5.121)$ using interval notation. Shade in the solution area on the graph above for $-2x^2+12x-13>-4$

If we want to find where $-2x^2+12x-13<-4$ we still use the inputs we already found. We notice that in order for the output to be less than -4 we must choose inputs that are less than .879 OR greater than 5.121. To write this statement symbolically, we write: $x < .879$ OR $x > 5.121$ using inequality notation or $(-\infty, .879) \cup (5.121, \infty)$ using interval notation. Notice that we can write one large inequality (or one interval if using that notation) when we have an AND statement, but need two separate inequalities (or intervals) when dealing with an OR statement.

Now you try it! Find all the values of the input for which $2x^2 + 12x - 13 < 5$ and then for which $2x^2 + 12x - 13 > 5$. Make sure you properly label your graph!

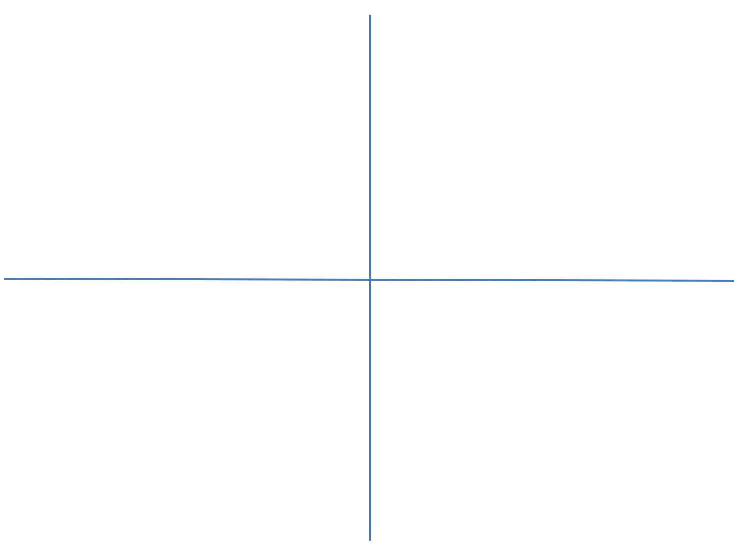


Let's try a couple with some real-life context!

A fastball is hit straight up over home plate. The ball's height, h (in feet), from the ground is modeled by: $h(t) = -16t^2 + 80t + 5$ where t is measured in seconds.

1. What is the maximum height of the ball above the ground?
2. At what time(s) after it is struck will the ball be 50 feet above the ground?
3. At what time(s) after it is struck will the ball be more than 50 feet above the ground?
4. At what times(s) after it is struck will the ball be less than 50 feet above the ground?

Think about what each question is asking before you begin.



An arrow is shot into the air. The arrow's height, h (in feet), above the ground can be modeled by: $h(t) = -16t^2 + 80t + 4$, where t is time since the arrow was shot in seconds.

1. What is the maximum height of the arrow above the ground? At what time does this occur?
2. At what time(s) after it is struck will the arrow be 70 feet above the ground?
3. At what time(s) after it is struck will the ball be more than 70 feet above the ground?
4. At what times(s) after it is struck will the ball be less than 70 feet above the ground?

Think about what each question is asking before you begin.