## **Transformations of Functions**

Transformations must be performed in the following order: Reflections, Stretches, Translations (RST)

	<b>Vertical</b> (Intuitive/The Way You Would Think)	Horizontal (Backwards/Counter-intuitive)
Reflections	$g(x) = -f(x)$ If $f(x) = x^2$ , then $g(x) = -x^2$ $f(x) = -x^2$ $\xrightarrow{\begin{array}{c} 8 & - \text{ f(x)} \\ 6 & - \\ -3 & -2 & -1 & 0 \\ -3 & -2 & -2 & -2 \\ -3 & -2 & -2 & -$	g(x) = f(-x) If $f(x) = \sqrt{x}$ , then $g(x) = \sqrt{-x}$ $\begin{bmatrix}3\\2\\1\\0\\0\\2&4&6\end{bmatrix} \rightarrow \begin{bmatrix}g(x)\\-6\\-6&-4\\-2&0\end{bmatrix}$ Horizontally reflected over the y-axis
Stretches	g(x) = 3f(x) If $f(x) =  x+2 $ , then $g(x) = 3 x+2 $ $f(x) = \frac{3}{4} + $	$g(x) = f(.5x)$ If $f(x) = \sin(x)$ , then $g(x) = \sin(.5x)$ $ \begin{bmatrix} 1 & f(x) \\ 0 & g(x) \\ 1 & g(x) \end{bmatrix} \rightarrow \begin{bmatrix} 1 & g(x) \\ 0 & g(x) \\ 1 & g(x) \end{bmatrix} $ $g(x)$ is NOT half, but "2 times as wide as" $f(x)$
Compressions	$g(x) = . \mathcal{F}(x)$ If $f(x) = \log x$ , then $g(x) = .5 \log x$ $\xrightarrow[0.5]{0.3} \qquad \rightarrow \qquad $	g(x) = f(3x) If $f(x) = x^3 + 5$ , then $g(x) = (3x)^3 + 5$ $\overbrace{\begin{array}{c}25 \\ -3 \\ -3 \end{array}}^{f(x)} \rightarrow \overbrace{\begin{array}{c}25 \\ -3 \\ -3 \end{array}}^{g(x)} is \text{ NOT 3 times, but "one - third as wide as" } f(x)$
Translations (Shifts)	g(x) = f(x) + 2 If $f(x) = \sqrt{x}$ , then $g(x) = \sqrt{x} + 2$ $\int_{a}^{5} + \frac{1}{2} + \frac$	$g(x) = f(x-3)$ If $f(x) =  x $ , then $g(x) =  x-3 $ $\int_{1}^{3} \int_{1}^{4} f(x)$ $\xrightarrow{1}{4 - 2 \ 0 \ 2 \ 4 \ 6}} \rightarrow \int_{1}^{3} \int_{1}^{3} \int_{1}^{4} f(x)$ $g(x) = f(x)$ is $f(x)$ shifted 3 units right $g(x) = f(x+5)$ If $f(x) = x^{2}$ , then $g(x) = (x+5)^{2}$ $\int_{1}^{4} \int_{1}^{2} \int_{1}^{4} \int$

## **Transformation Examples**

 $y = e^{-x}$ 

indicates that the graph was *Horizontally Reflected* over the y-axis. The -



 $g(x) = 2(x+5)^2$ 

indicates that the graph was *Vertically Stretched* by a *factor of 2* (twice as tall). The **2** The +5 indicates that the graph was *Horizontally Shifted* by 5 units to the left.

$$h(x) = -\sqrt{4x} + 3$$

- The indicates that the graph was *Vertically Reflected* over the x-axis.
- The **4** indicates that the graph was <u>Horizontally Compressed</u> by a **factor of .25** (1/4 as wide).

The +3 indicates that the graph was <u>Vertically Shifted</u> by 3 units up.

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$$\begin{array}{c} 3 \\ 0 \\ -3 \end{array} \xrightarrow{0} 4 \hspace{0.1cm} 8 \end{array} \hspace{0.1cm} \rightarrow \hspace{0.1cm} \begin{array}{c} 3 \\ 0 \\ -3 \end{array} \xrightarrow{0} 4 \hspace{0.1cm} 8 \end{array} \hspace{0.1cm} \rightarrow \hspace{0.1cm} \begin{array}{c} 3 \\ 0 \\ -3 \end{array} \xrightarrow{-\sqrt{4x}} \xrightarrow{-\sqrt{4x}+3} \\ -3 \end{array} \hspace{0.1cm} \rightarrow \hspace{0.1cm} \begin{array}{c} 3 \\ 0 \\ -3 \end{array} \xrightarrow{-\sqrt{4x}+3} \xrightarrow{-\sqrt{4x}+3} \\ -3 \end{array} \hspace{0.1cm} \rightarrow \hspace{0.1cm} \begin{array}{c} 3 \\ 0 \\ -3 \end{array} \xrightarrow{-\sqrt{4x}+3} \xrightarrow{-\sqrt{4x}+3} \\ -3 \end{array} \hspace{0.1cm} \rightarrow \hspace{0.1cm} \begin{array}{c} 3 \\ 0 \\ -3 \end{array} \xrightarrow{-\sqrt{4x}+3} \xrightarrow{-\sqrt{4x}+3}$$

$$g(x) = -3\sin\left(2\left(x - \frac{\pi}{2}\right)\right) + 6$$

$$f(x) = \sin x$$

$$\xrightarrow{f(x) = \sin x}$$

$$\xrightarrow{f(x) = -3\sin x}$$

The PERIOD has been reduced to  $\pi$