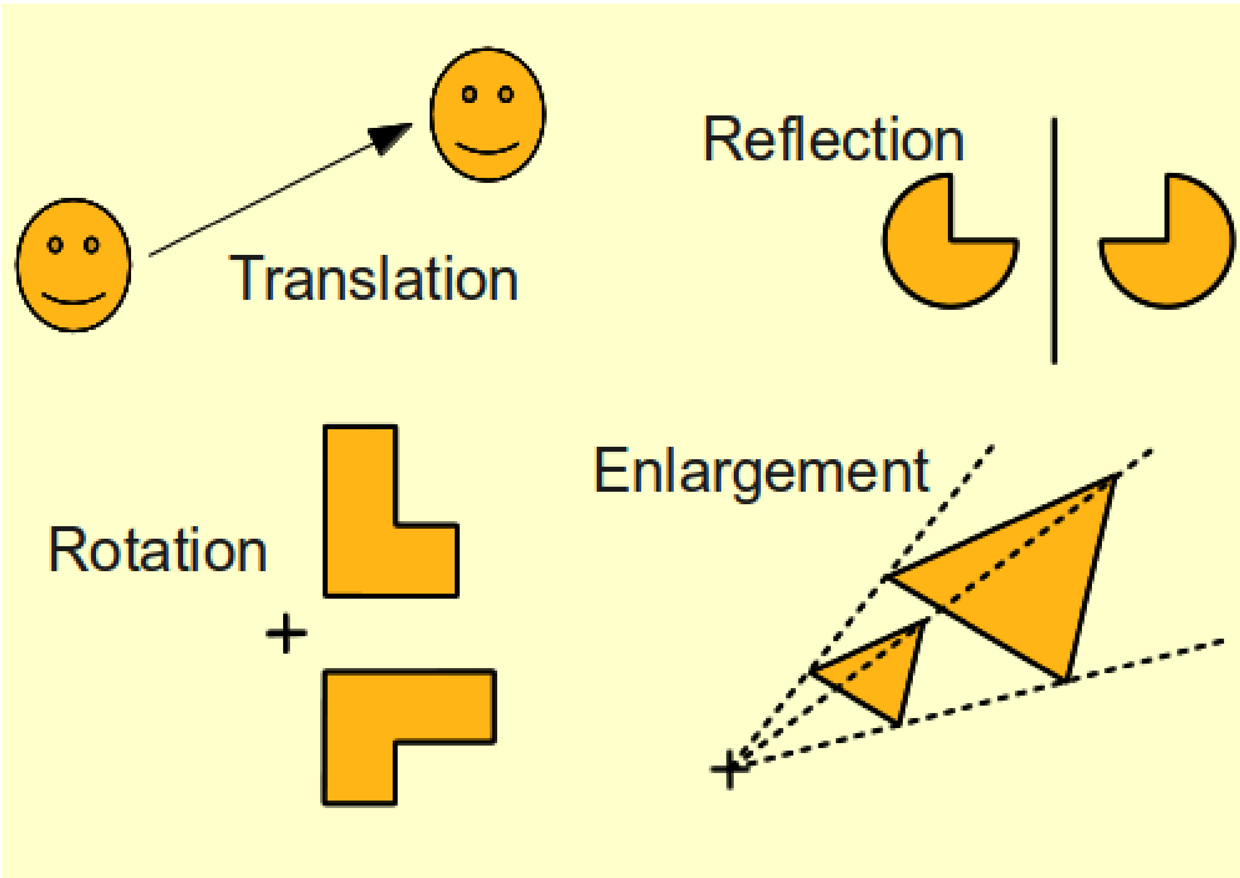
Transformations of Functions Workshop



CHANDLER-GILBERT
COMMUNITY COLLEGE

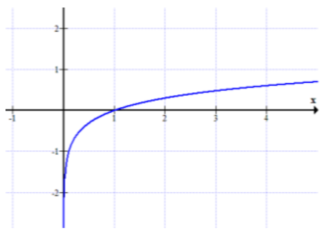
Learning Center

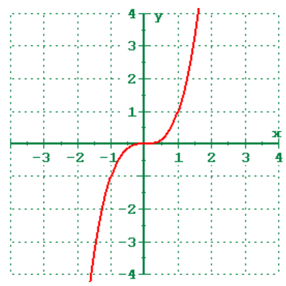
Overview

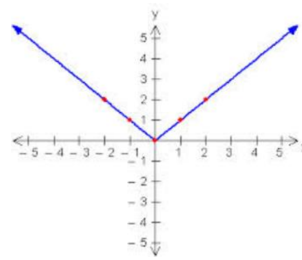
- Review parent functions, their tables, and their graphs
- Reflections……………Vertically and Horizontally
- Stretches………………Vertically and Horizontally
- Compressions…………Vertically and Horizontally
- Translations (Shifts)…..Vertically and Horizontally

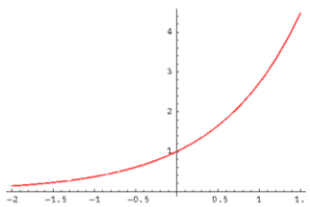
Review parent functions, their tables, and their graphs

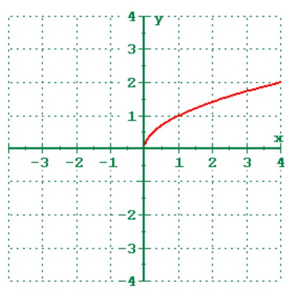
Before you start learning Transformation of Functions, you should be familiar with the following parent functions and their graphs: $f(x) = x^2, f(x) = x^3, f(x) = |x|, f(x) = \sqrt{x},\ f(x) = \ 2^x$, and $f(x) = \log x$. Identify each of these functions below:

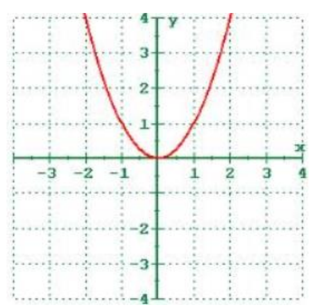












Transformations of Functions

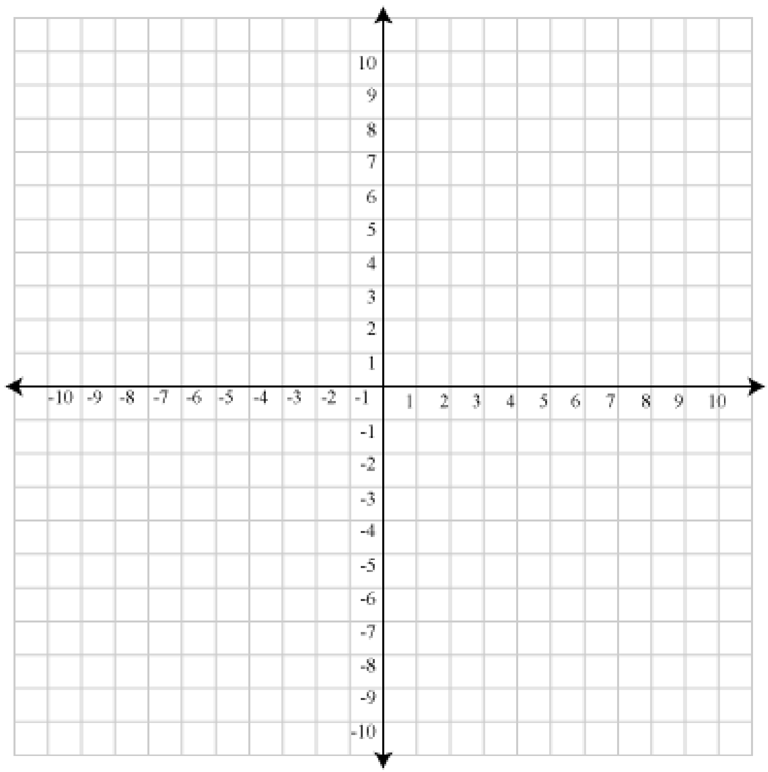
Once you have a good understanding about the previous parent functions, you can be introduced to the following Transformations of Functions the form: $g(x) = a \cdot f\big(b(x - c)\big) + d$.

Reflections

Case I: Applying the parent function $f(x) = x^2$, explore how $f(x)$ is transformed into a function $g(x)$ by multiplying the output values of $f(x)$ by -1. Fill in the blanks below and sketch the graphs of both the parent function and the transformed function using the same set of axes.

a) $g(x) = \_\_f(x) = \_\_\_x^2$

x	$f(x)$	$g(x)$
-3	9	
-2	4	
-1	1	
0	0	
1	1	
2	4	
3	9	



How does changing the sign of the function $f(x)$ affect its graph? Why?

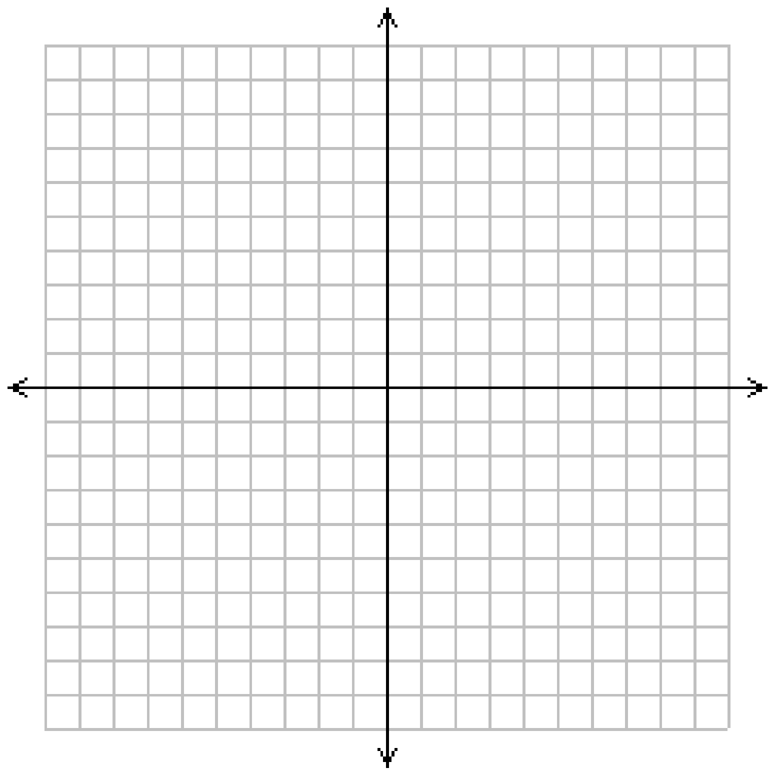
Case II. Using the parent function $f(x) = \sqrt{x}$, explore how the function $f(x)$ is transformed into a function $h(x)$ by multiplying its input values by -1. Fill in the blanks below and sketch the graph of both the parent function and the transformed function.

$h(x) = f(-x) = \sqrt{-x}$

x	$f(x)$
-16	-----
-9	------
-4	------
-1	------
0	0
1	1
4	2
9	3
16	4

x	$h(x)$

	0
	1
	2
	3
	4



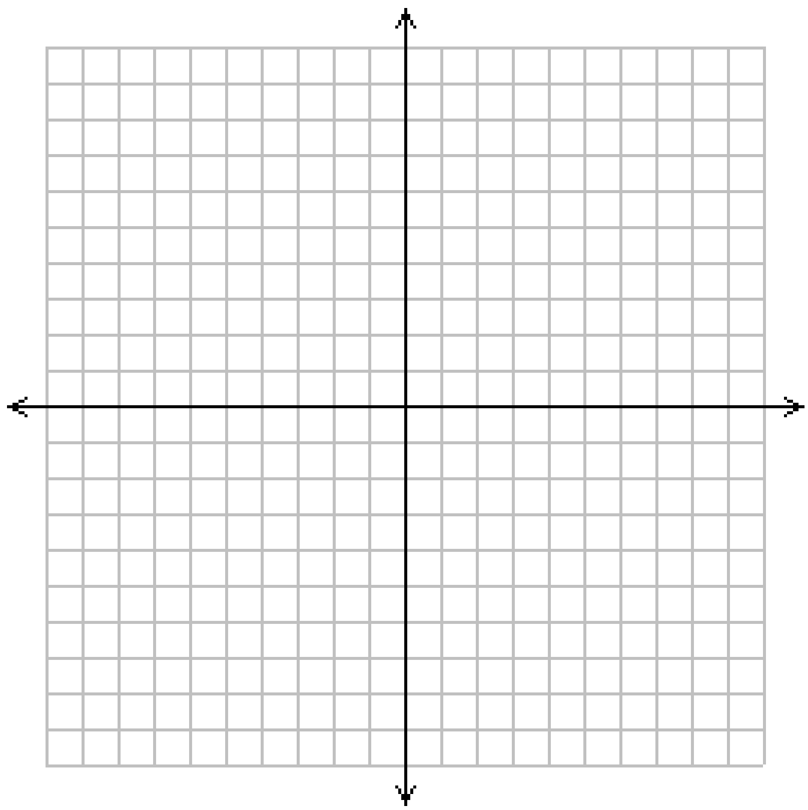
How does changing the sign of the input values x of the function $f(x)$ affect its graph?

Stretches

Case I: Using the parent function $f(x) = |x|$, explore how the function $f(x)$ is transformed into a function $g(x)$ when multiplying f(x) by a real number a>1. Fill in the blanks below and sketch the graphs of both the parent function and the transformed function.

$g(x) = \_\_ \cdot f(x) = \_\_\_ \cdot |x|$

x	$f(x)$	$g(x)$
-5	5	
-4	4	
-3	3	
-2	2	
-1	1	
0	0	
1	1	
2	2	
3	3	
4	4	
5	5	



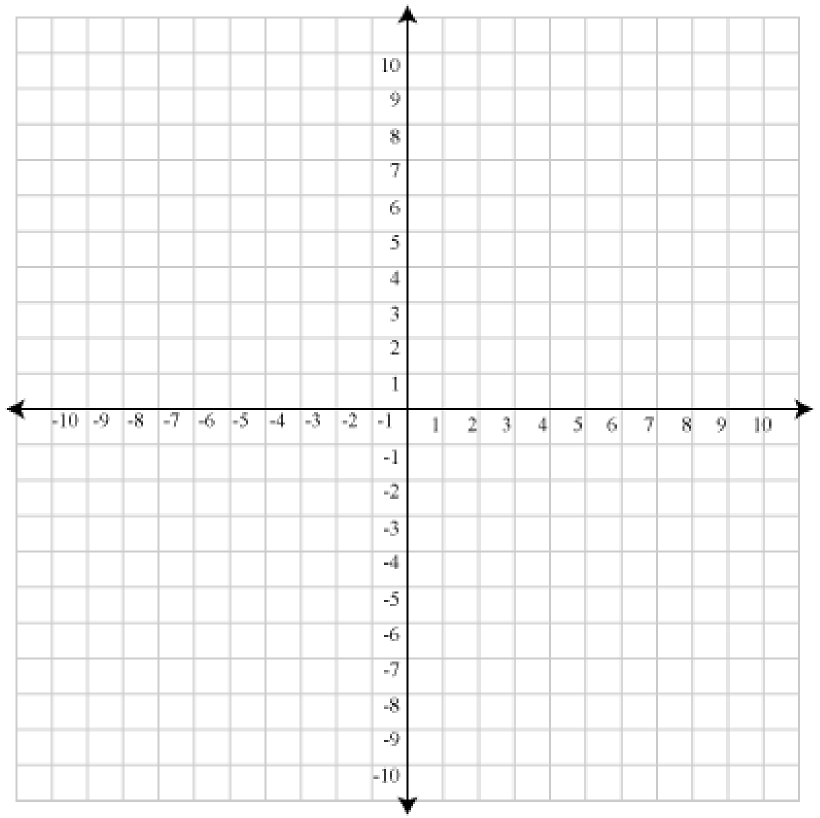
How does multiplying the function f(x) *by a* real number a>1 affect its graph?

Case II. Using the parent function $f(x) = x^2$, explore how $f(x)$ is transformed into a function $h(x)$ when multiplying the input value x of the function f(x)by a real number b, 0<b <1. Fill in the blanks below and sketch the graphs of both the parent function and the transformed function.

$h(x) = f(\_\_\_x) = (\_\_\_x)^2$

x	$f(x)$
-2	4
-1	1
0	0
1	1
2	4

x	$h(x)$
	4
	1
	0
	1
	4



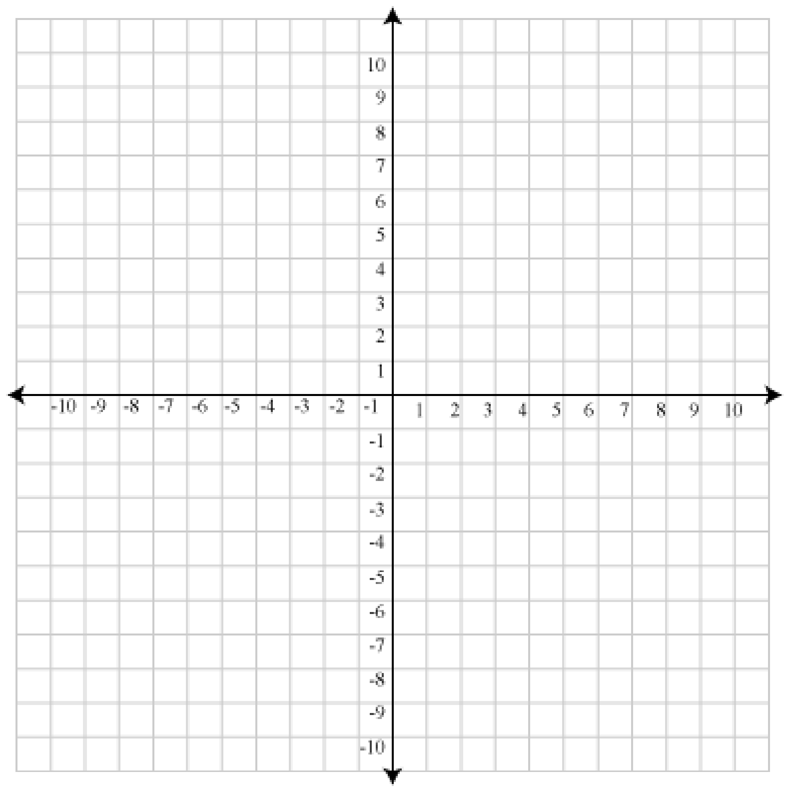
How does multiplying the input x of the function f(x) by a real number b, 0<b <1, affect its graph?

Compressions

Case I: Using the parent function $f(x) = log x$, explore how $f(x)$ is transformed into a function $g(x)$ when multiplying f(x)by a real number a, where $0<a<1$. Fill in the blanks below and sketch the graph of both the parent function and the transformed function.

$g(x) = \_\_ \cdot f(x) = \_\_\_ \cdot log x$

x	$f(x)$	$g(x)$
0	---	
1	0	
2	0.30	
3	0.48	
4	0.60	
5	0.70	
6	0.77	
7	0.85	
8	0.9	
9	0.95	
10	1	



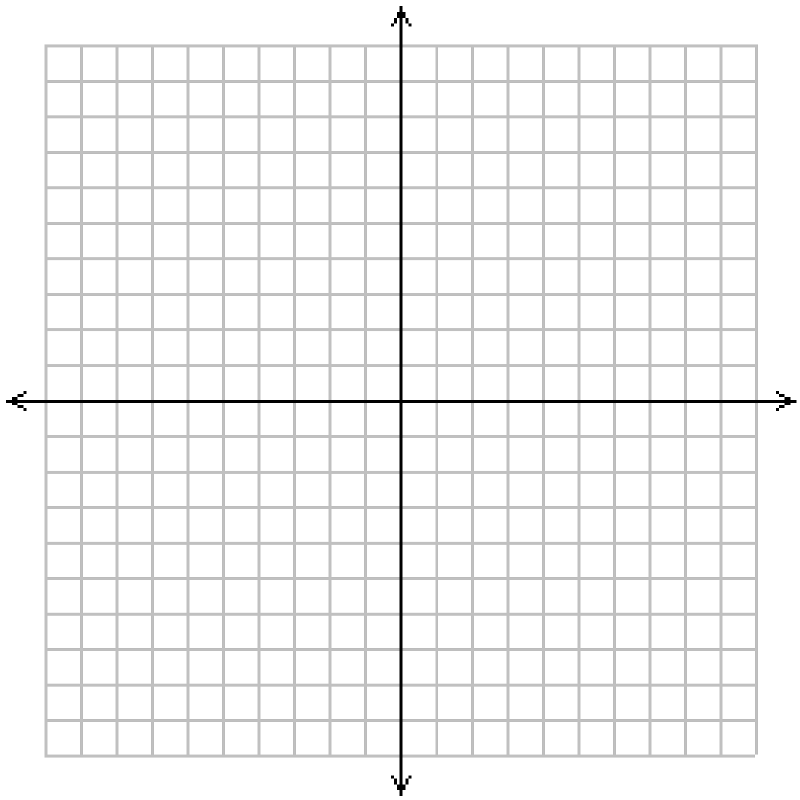
How does multiplying the function f(x) by a real number a**,** where $0<a< 1$, affect its graph?

Case II. Using the parent function$f(x) = x^3$, explore how $f(x)$ is transformed into a function $h(x)$ when multiplying the input value x of the function $f(x)$by a real number $b > 1$. Fill in the blanks below and sketch the graphs of both the parent function and the transformed function.

$h(x) = f(\_\_\_ \cdot x) = (\_\_\_\_ \cdot x)^3$

x	$f(x)$
-3	-27
-2	-8
-1	-1
0	0
1	1
2	8
3	27

x	$h(x)$
	-27
	-8
	-1
	0
	1
	8
	27



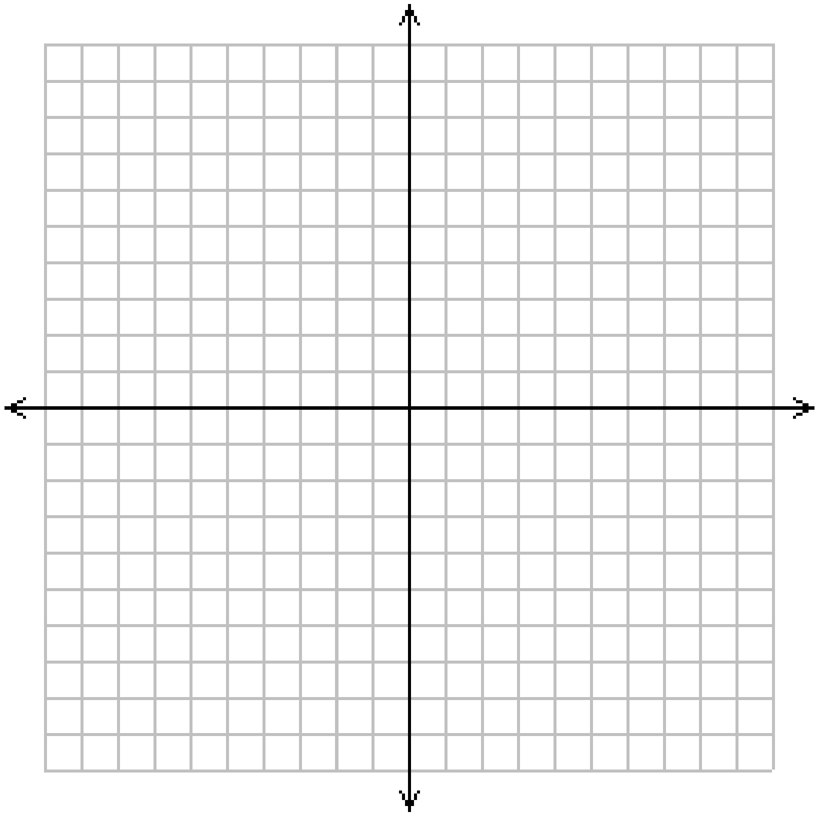
How does multiplying the input x of the function f(x) by a real number b, b >1 affects its graph?

Translations (Shifts)

Case I. Applying the parent function $f(x) = \sqrt{x}$, explore how $f(x)$ is transformed into a function $g(x)$ when adding a real number d > 0 to the output values of $f(x)$. Fill in the blanks below and sketch the graphs of both the parent function and the transformed function.

$g(x) = f(x) +$ ____ $= \sqrt{x}$ + ____

x	$f(x)$	$g(x)$
0	0	
1	1	
4	2	
9	3	
16	4	
25	5	
36	6	

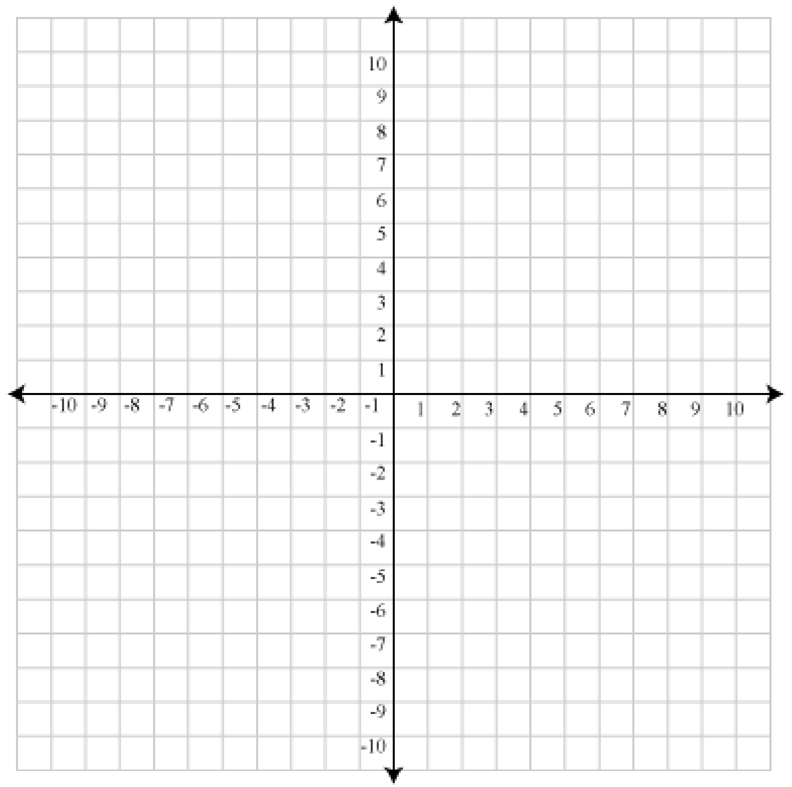


How does adding a real number d > 0 to the output values of the parent function $f(x)$ affect its graph, that is, what can we say about the graph $g(x)$?

Case II. Using the parent function $f(x) = |x|$, explore how $f(x)$ is transformed into a function $j(x)$ when subtracting a real number d > 0 from the output values of $f(x)$. Fill in the blanks below and sketch the graph of both the parent function and the transformed function.

$j(x) = f(x) - \_\_\_\_ = |x| \; - \; \_\_\_\_$

x	$f(x)$	$j(x)$
-7	7	
-6	6	
-5	5	
-4	4	
-3	3	
-2	2	
-1	1	
0	0	
1	1	
2	2	
3	3	
4	4	
5	5	
6	6	
7	7	



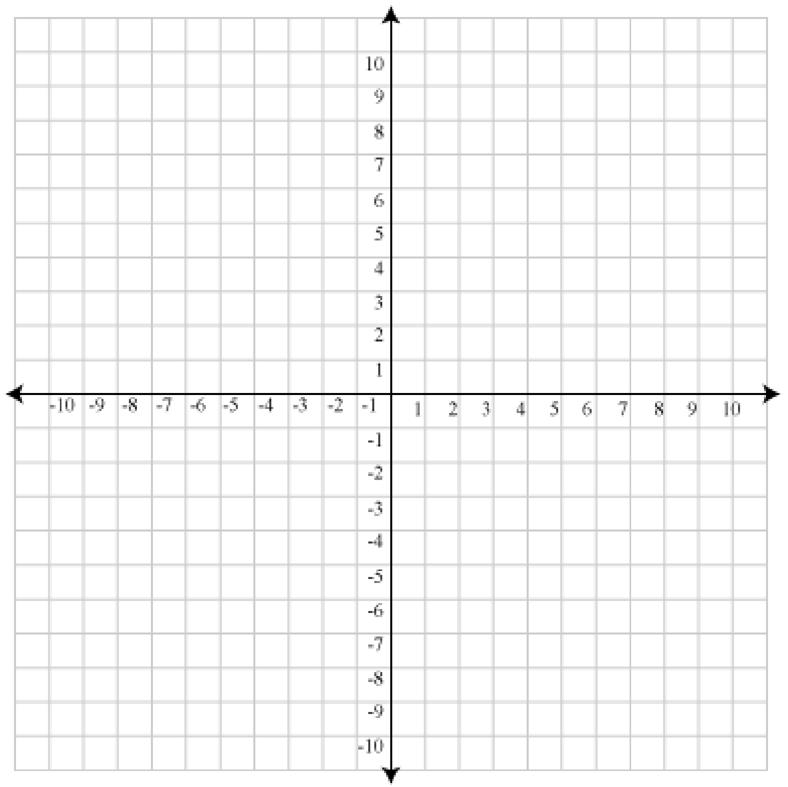
How does subtracting a number d > 0 from the output values of the parent function $f(x)$ affect its graph? That is, what can we say about the graph of $j(x)$?

Case III. Using the parent function $f(x) = |x|$, explore how $f(x)$ is transformed into a function $h(x)$ when adding a real number c > 0 to the input x of $f(x)$. Fill in the blanks below and sketch the graph of both the parent function and the transformed function.

$h(x) = f(x + \_\_\_) = |x + \_\_\_\_|$

x	$f(x)$
-4	4
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3
4	4

x	$h(x)$
	4
	3
	2
	1
	0
	1
	2
	3
	4



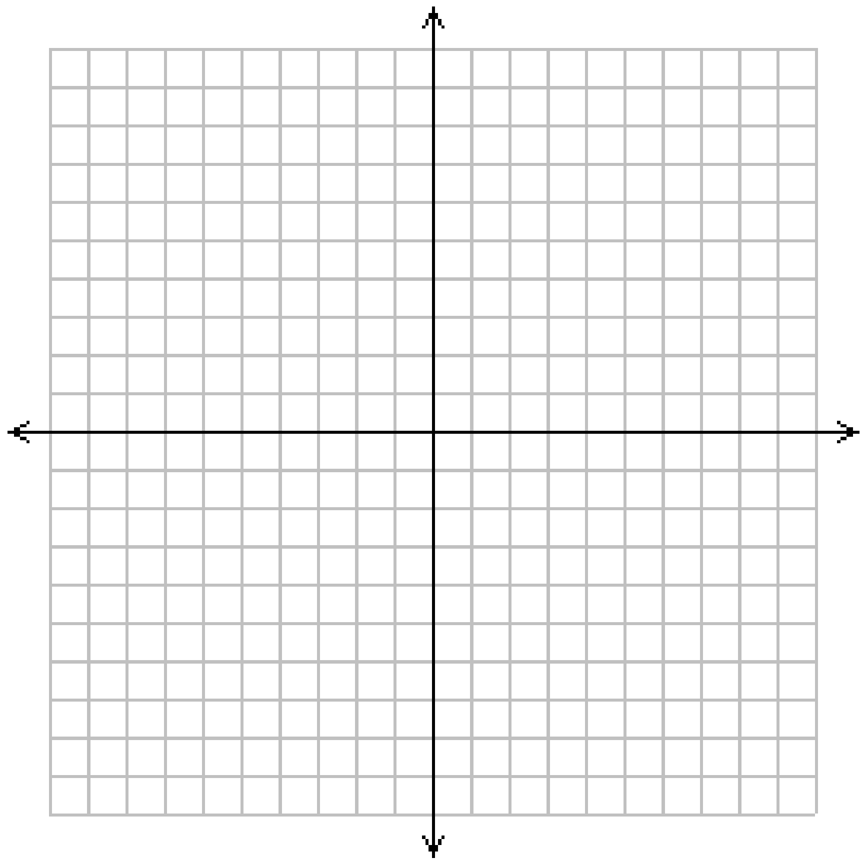
How does adding the constant c > 0 to the input x of the parent function $f(x)$ affect its graph; that is, what can we say about the graph of $h(x)$?

Case IV. Using the parent function$f(x) = x^2$, explore how $f(x)$ is transformed into a function $k(x)$ when subtracting a real number c > 0 from the input x of $f(x)$. Fill in the blanks below and sketch the graph of both the parent function and the transformed function.

$k(x) = f(x - \_\_\_) = (x - \_\_\_\_)^2$

x	$f(x)$
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

x	$k(x)$
	9
	4
	1
	0
	1
	4
	9



How does subtracting a constant c > 0 from the input x of the parent function $f(x)$ affect its graph; that is, what can we say about the graph $k(x)$?

Practice Problems

Part I. Describe each of the following transformation of $f(x)$ in the correct order to create the transformed function $g(x) = a \cdot f\big(b(x-c)\big) + d$

Part II. Identify and describe the transformations applied to the parent function $f(x) = x^3$ to produce each of the transformed function below:

1. $h(x) = f(x) - 5$

2. $i(x) = f(x-1) + 3$

3. $j(x) = -f(3x) - 1$

4. $k(x) = -\frac{1}{4}(x+2)^3 - 3$

5. $l(x) = (-\frac{1}{5}x)^3 + 1$

Part III. Create the table of values and sketch the graph for each of the transformed functions from part II relating them to the parent function $f(x)$.

Answers to the Practice Problems

Part I.

	Vertical	Horizontal
Factor	a	b
Shift	d	c
	Affects outputs	Affects inputs

Part II.

1. The graph of $h(x)$ is the graph of $f(x)$shifted vertically 5 units downward.

2. The graph of the graph of $i(x)$ is $f(x)$ shifted right 1 unit and up 3 units.

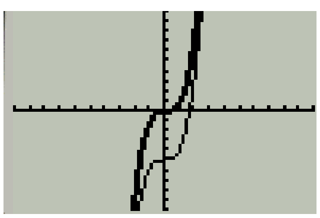
3. $j(x)$ is $f(x)$ vertically reflected over the x-axis, compressed horizontally by a factor of 1/3, and then shifted down 1 unit.

4. The graph of k(x) is the graph of $f(x)$ vertically reflected over the x-axis, vertically compressed by a factor of ¼, shifted left 2 units, and shifted down 3 units.

5. The graph of $l(x)$ is the graph of $f(x)$horizontally reflected over the y-axis, stretched horizontally by a factor of 5 units, and then shifted up 1 unit.

Part III.

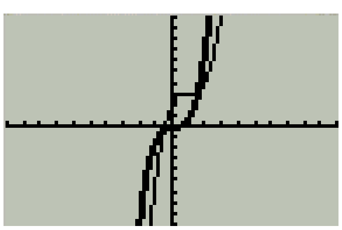
1. The thicker graph represents $f(x) = x^3$ and the other graph represents $h(x) = f(x) - 5$



x	$f(x)$	$h(x)$
-2	-8	-13
-1	-1	-6
0	0	-5
1	1	-4
2	8	3

Vertical translation affects the outputs

2. The thicker graph represents $f(x) = x^3$ and the other graph represents $i(x) = f(x - 1) + 3$



x	$f(x)$
-2	-8
-1	-1
0	0
1	1
2	8

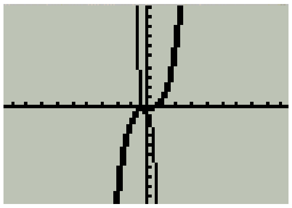
x	$f(x-1)$
-1	-8
0	-1
1	0
2	1
3	8

Shift right 1 affects inputs

x	$i(x)$
-1	-5
0	2
1	3
2	4
3	11

Shift up 3 affects outputs

3. The thicker graph represents $f(x) = x^3$ and the other graph represents $j(x) = -f(3x) - 1$



x	$f(x)$
-2	-8
-1	-1
0	0
1	1
2	8

x	$-f(3x)$
-2	8
-1	1
0	0
1	-1
2	-8

Vertical Reflection affects the outputs

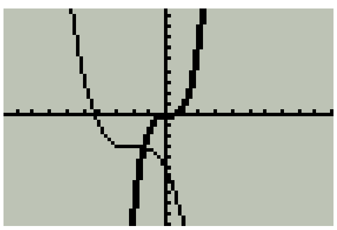
x	$-f(3x)$
-2/3	8
-1/3	1
0	0
1/3	-1
2/3	-8

Horizontal compression affects the inputs

x	$j(x)$
-2/3	7
-1/3	0
0	-1
1/3	-2
2/3	-9

Vertical translation affects the outputs

4. The thicker graph represents $f(x) = x^3$ and the other graph represents $k(x) = -\frac{1}{4}(x+2)^3 - 3$



x	$f(x)$
-2	-8
-1	-1
0	0
1	1
2	8

x	$-f(x)$
-2	8
-1	1
0	0
1	-1
2	-8

Vertical reflection affects outputs

X	$-\frac{1}{4}f(x)$
-2	2
-1	¼
0	0
1	-1/4
2	-2

Vertical Compression affects outputs

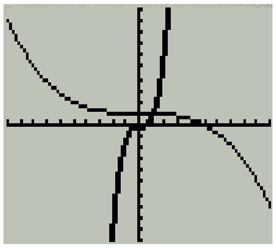
x	$-\frac{1}{4}f(x+2)$
-4	2
-3	¼
-2	0
-1	-1/4
0	-2

Horizontal translation affects inputs

x	$k(x)$
-4	-1
-3	$-2\frac{3}{4}$
-2	-3
-1	$-3\frac{1}{4}$
0	-5

Vertical translation affects outputs

5. The thicker graph represents $f(x) = x^3$ and the other graph represents $l(x) = (-\frac{1}{5}x)^3 + 1$



x	$f(x)$
-2	-8
-1	-1
0	0
1	1
2	8

x	$f(-x)$
2	-8
1	-1
0	0
-1	1
-2	8

Horizontal reflection affects inputs

x	$f\left(-\frac{1}{5}x\right)$
10	-8
5	-1
0	0
-5	1
-10	8

Horizontal stretch affects the inputs

x	$l(x)$
10	-7
5	0
0	1
-5	2
-10	9

Vertical translation affects the outputs